An abstract of the results of my research

I mainly have been studying the derived categories and the equivalences between them for these two years. Recently, it turns out that they play important roles in Lie theory. By the way, Rickard's works are most important to study them: In 1989, Rickard gave necessary and sufficient conditions for two derived categories of module categories to be equivalent, which is called Rickard's theorem.

As is well-known that $\operatorname{Mod}\Lambda$ and $\operatorname{Mod}\Gamma$ are Morita equivalent if and only if there exists a progenerator P in $\operatorname{Mod}\Lambda$ such that $\operatorname{End}_\Lambda(P) \cong \Gamma$. On the other hand, according to Rickard's theorem, two rings Λ and Γ are said to be derived equivalent if there exists a tilting complex $T^{\cdot}$ for Λ such that $\operatorname{End}_{\mathcal{K}(\operatorname{Mod}\Lambda)}(T^{\cdot}) \cong \Gamma$. Thus Rickard generalized the concept of progenerators to that of tilting complexes.

Rickard's theorem has many applications: For example, it is applied to study Broué's abelian defect group conjecture and plays an important role to prove derived equivalences in the classification of self-injective algebras. If two self-injective algebras are derived equivalent, then by Rickard's theorem, they are stably equivalent. Notice that to show the existence of derived equivalences is easier than that of stable equivalences. Also, according to Rickard's theorem, it turns out that the center of a ring, the Grothendieck group of a ring, the Hochschild cohomology group of an algebra and the representation dimension between self-injective algebras are invariant under derived equivalences.

It follows from the above that it is interesting to study the conditions for two derived categories to be equivalent.

My master's thesis started with an account on Rickard's theorem. This is because I thought that Rickard's original paper was very hard to read it. Indeed, I believe that I have given a very easier approach to his theorem by using the knowledge on the derived category.

Second purpose is to give a review of tilting theory from the point of derived categories by using Rickard's theorem. Let A be a finite-dimensional algebra over an algebraically closed field. If T is a tilting module over A, then it is known that A and $B = \operatorname{End}_A(T)$ have module categories which have many close relations, not necessarily being equivalent. In particular, it turns out that the Grothendieck groups of A and of B are isomorphic. They form important parts in tilting theory. Now, it is almost clear that as stalk complexes, tilting modules are tilting complexes. Therefore, Rickard's theorem allows us to see tilting theory from the viewpoint of derived categories.

There has been a problem whether tilting stalk complexes give conversely tilting modules. I have given an affirmative answer to this problem.