Abstract of the results of my research

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I mainly have been studying the derived categories and the equivalences between them for these three years. Recently, it turns out that they play important roles in Lie theory. By the way, Rickard's works are most important to study them: In 1989, Rickard gave necessary and sufficient conditions for two derived categories of module categories to be equivalent, which is called Rickard's theorem.

Rickard's theorem is a generalization of Morita's theorem on Morita equivalences. Namely, Rickard generalized the concept of progenerators to that of tilting complexes: The progenerators define Morita equivalences and the tilting complexes define derived equivalences.

Rickard's theorem has many applications: For example, it plays an important role to study Broué's abelian defect group conjecture. By the way, it is possible to give a review of tilting theory from the point of view of derived categories by using Rickard's theorem. This was a purpose of my master's thesis. Let A be a finite-dimensional algebra over an algebraically closed field. If T is a tilting module over A, then it is known that A and $B = \mathrm{End}_A(T)$ have module categories which have many close relations, not necessarily being equivalent. In particular, it turns out that the Grothendieck groups of A and of B are isomorphic, and that A has finite global dimension if and only if so does B. These facts form important parts in tilting theory. Now, it is almost clear that as stalk complexes, tilting modules are tilting complexes. Therefore, Rickard's theorem allows us to see tilting theory from the point of derived categories. Also, there has been a problem whether tilting stalk complexes give conversely tilting modules. I have given an affirmative answer to this problem.

On the other hand, it is known that a tilting sheaf $\mathcal{T}$ on a smooth projective variety $\mathbf{X}$ over an algebraically closed field defines an equivalence between derived categories of the categories of coherent sheaves on $\mathbf{X}$ and of finitely generated $B = \mathbf{Mod}_{\mathbf{X}}(\mathcal{T}, \mathcal{T})$-modules (Beilinson's lemma). Thus tilting sheaves can be regarded as tilting stalk complexes. There, however, exists a difference between them: the definition of tilting sheaves requires the finiteness of the global dimension of B. Here, it is natural to pose the question why there exists the difference. Since tilting stalk complexes give tilting modules, I try to give an answer by considering objects corresponding to tilting modules.

More generally, I investigate the problem in Grothendieck categories. This is natural because the category $\mathrm{Qco}\,\mathbf{X}$ of quasi-coherent sheaves on $\mathbf{X}$ and the module category are Grothendieck categories. Let $\mathcal{A}$ be a Grothendieck category. I have defined n-tilting stalk complexes and n-tilting objects on $\mathcal{A}$. And n-tilting objects correspond to tilting modules. Moreover, I have shown that if the derived category $\mathcal{D}(\mathcal{A})$ of $\mathcal{A}$ is compactly generated, then $T \in \mathcal{A}$ is a 1-tilting stalk complex if and only if it is a 1-tilting object and is compact as an object of $\mathcal{D}(\mathcal{A})$. Here, 1-tilting objects were defined by Colpi in 1999. And Colpi showed some basic facts on tilting theory in Grothendieck categories. To extend these results to an arbitrary positive integer n, I reconstructed the tilting theory, except for the statement on Grothendieck groups, by using techniques of derived categories.

After this, I plan to apply my results to tilting sheaves.