Research Plan

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1. Problems

I shall study the following problems:

(1) Evaluate the number of holomorphic sections of holomorphic families of Riemann surfaces.

(2) Show the global non-triviality of the above triple $(\mathcal{M}, \pi, R)$.

2. Plans

(1) First, by using examples of holomorphic families, I shall work on evaluating the number of holomorphic sections of them. Next, I will conjecture the number of holomorphic sections。 It is known that every holomorphic family of Riemann surface and holomorphic sections of it are determined by the monodromy of the holomorphic family. （Imayoshi ＆ Shiga's Rigidity Theorem） Thus, if I decide the monodromy, then I can estimate the number of holomorphic sections. And by use of 2,3 dimensional hyperbolic geometries and the theory of Kleinian groups, I can determine the monodromy.

(2) Kodaira surface, which is a holomorohic family constructed by Kodaira, is showed to be locally non-trivial. And $(\mathcal{M}, \pi, R)$ is also showed to be locally non-trivial. Then, is $(\mathcal{M}, \pi, R)$ globally non-trivial? We give a defining equation of it by using algebraic equations. So First, I shall study this problem with the defining equation.

At the same time, given two sets of six distinct points on the Riemann sphere, I study when the two sets are mapped to each other under a Mobius transformation. Since each fiber of $(\mathcal{M}, \pi, R)$ is a closed Riemann surface of genus two, it is represented as a two-sheeted branched covering surface of the Riemann sphere branched over six points. It is well known two fibers S and S' are biholomorphically equivalent if and only if there is a Mobius transformation which takes the set of branch points of S to the set of branch points of S'. By use of the theory of configuration spaces, we can study the problem.