Research plan

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Supposing one fact, we can prove the Bala-Carter theorem in good characteristic. The fact is the following:

Theorem 3. Suppose that p is good for G. For any nilpotent element X in $\mathfrak{g}$, we have $m(X) \leq 2$.

This fact can be proved by the Bala-Carter theorem. If the characteristic p of the base field of G is more than $3h - 2$, we can prove using a part of the Bala-Carter's method, where h is the coxeter nunmber. However we cannot find the proof of the theorem 3 for arbitrary good p, not using the Bala-Carter theorem. When we prove Theorem 3, one can show that we may assume X is distinguished.

On the other hand, the existence of good transverse slices to nilpotent orbits can be proved, if we don't prove Theorem 3. However it is not convenient to use.

The representation theory of algebraic groups is developing and exists a lot of room for improvement. We will study this theory deeper, and reseach new problems. If I have a chance, I want to prove theorem 3, and find a direct proof of the finiteness of numbers of the nilpotent orbits for bad primes.