Summary of my research

Takehisa Tsujii

In 2003, the proof of the Bala-Carter theorem given by A. Premet was epoch-making. If the characteristic p of the base field is good, this theorem was proved by Pommerening, first. But its proof needed case-by-case consideration through the classification of almost simple algebraic groups, and type E were ommited. On the other hand, Premet's proof was a noncomputational proof, and simultaneously proved the existence of good transverse slices to the nilpotent orbits. However it needed some big facts. So we wonder there exists a briefly and easier proof than Premet's proof. In the reseach, we can find a new fact on the Kempf-Rousseau theory, which was used in Premet's proof.

Theorem 1. *Let $\rho\colon G \to GL(V)$ a finite dimensional rational representation of a connected reductive algebraic group G over an algebraically closed field. Let $v \in V \setminus 0$ be G-unstable, and λ an optimal cocharacter for v. Write $v = \sum_{i \geq k} v_i$, $v_i \in V(i;\lambda)$ the decomposition by λ ($v_k \neq 0$). Then the set of optimal cocharacters for v_k is equal to those for v.*

We were going to announce this fact in my master's thesis, but the proof has a fault, so we cannot announce in the thesis. However we can prove the thereom completely, later. The proof of this theorem is simple, only using the Kempf-Rousseau theory and basic facts of algebraic groups. This yields the following:

Corollary. *Consider the adjoint representation of G. Let X be a nilpotent element in the Lie algebra $\mathfrak{g}$ of G, λ_X the set of primitive optimal cocharacters for X. For any $\lambda \in \Lambda_X$,*

$$m(X, \lambda) = \min\{n \in \mathbb{Z} | v_n \neq 0\}.$$

is uniquely determined, say $m(X) = k$. Suppose that p is good for G. Then there exists $\lambda \in \Lambda_X$ such that $X \in \mathfrak{g}(k;\lambda)$.

This corollary yields several facts on nilpotent elements, using techniques for Bala and Carter, and Premet. We can prove the following important result:

Theorem 2. *Suppose that p is good for G. Then the centralizer of any nilpotent element has a natural Levi decomposition.*