Research Plan

The differential operator ${}^t f(\partial)$ determining the b-function of an invariant f on a flag manifold G/P is defined by a universal Verma module. This module is defined similarly to a usual Verma module by a scalar extension $R \otimes U(\mathfrak{g})$ of the enveloping algebra $U(\mathfrak{g})$ of $\mathfrak{g} = \mathrm{Lie}(G)$. Here R is a polynomial ring. For invariants on G/P, ideals of R are defined by universal Verma modules. The generators of these ideals correspond to b-function.

In the case of the quantum analogue a universal Verma module is defined similarly. Then the coefficient ring R is a Laurent polynomial ring over the rational function field $\mathbb{C}(q)$. By the similar way "quantum differential operators" are defined, and quantum deformations of b-functions correspond to generators of ideals of R. I will research "quantum differential operators".

The b-function of the basic relative invariant f of the prehomogeneous vector space of commutative parabolic type of type A_{2n-1} is calculate by the Capelli identity. The Capelli identity is the following:

$$\det(x_{ij})_{1\leq i,j\leq n} \det(\frac{\partial}{\partial x_{ij}})_{1\leq i,j\leq n} = \det\left(\sum_{k}^{n} x_{ki}\frac{\partial}{\partial x_{kj}} + (n-j)\delta_{ij}\right)_{1\leq i,j\leq n}.$$

The right hand side of the Capelli identity is in center of $U(\mathfrak{gl}_n)$. The left hand side is coincide with the operator $f\,{}^t f(\partial)$. I will study the central element corresponding to this operator $f\,{}^t f(\partial)$ in terms of universal Verma modules. Moreover I will study the existence of corresponding central elements and Capelli identity in the case of the quantum analogue. Since the unit of R is not only scalar in this cases, there is the question whether such central elements and Capelli identity depend on the choice of generator of ideal of R. From this correspondence I want to get the new expression and properties of central elements of quantized enveloping algebras. I hope that the good relation between the center of the quantized enveloping algebra and "quantum differential operators".

I also have an interest in the space consisting of "quantum differential operators". Can this space be constructed in general? In the classical case, that is $q = 1$, differential operators are generated by $\frac{\partial}{\partial x_i}$ and x_j, where $\{x_i\}$ is a coordinate system. In the quantum case are there the generators corresponding to $\frac{\partial}{\partial x_i}$? I want to investigate the structure of the space of "quantum differential operators".

Furthermore, I will construct "quantum differential operators" on a quantum analogue of a general algebraic variety with an action of an algebraic group except for G/P.