

Research Plan

The differential operator ${}^t f(\partial)$ determining the b -function of an invariant f on a flag manifold G/P is defined by a universal Verma module. This module is defined similarly to a usual Verma module by a scalar extension $R \otimes U(\mathfrak{g})$ of the enveloping algebra $U(\mathfrak{g})$ of $\mathfrak{g} = \text{Lie}(G)$. Here R is a polynomial ring. For invariants on G/P , ideals of R are defined by universal Verma modules, and the generators of these ideals correspond to b -function.

In the case of the quantum analogue a universal Verma module is defined similarly. Then the coefficient ring R is a Laurent polynomial ring over the rational function field $\mathbb{C}(q)$. By the similar way “quantum differential operators” are defined, and quantum deformations of b -functions correspond to generators of ideals of R . I will research “quantum differential operators”. For the prehomogeneous vector space (L, \mathfrak{m}^+) of commutative parabolic type associated with the simple Lie algebra of type A_{2n-1} , Noumi, Umeda and Wakayama were construct “quantum differential operators” by the R -matrix, and gave the quantum deformation of the b -function. I will consider the construction of R -matrices from “quantum differential operators” defined by universal Verma modules.

The b -function of the basic relative invariant f of (L, \mathfrak{m}^+) of type A_{2n-1} is calculate by the Capelli identity. The Capelli identity is the following:

$$\det(x_{ij})_{1 \leq i, j \leq n} \det\left(\frac{\partial}{\partial x_{ij}}\right)_{1 \leq i, j \leq n} = \det\left(\sum_k^n x_{ki} \frac{\partial}{\partial x_{kj}} + (n-j)\delta_{ij}\right)_{1 \leq i, j \leq n}.$$

The right hand side of the Capelli identity is in center of $U(\mathfrak{gl}_n)$. The left hand side is coincide with the operator ${}^t f(\partial)$. I will study the central element corresponding to this operator ${}^t f(\partial)$ in terms of universal Verma modules. Moreover I will study the existence of the corresponding central elements and the Capelli identity in the case of the quantum analogue. Since the unit of R is not only scalar in this cases, there is the question whether such central elements and the Capelli identity depend on the choice of generator of ideal of R . From this correspondence I want to get the new expression and properties of central elements of quantized enveloping algebras. I hope that the good relation between the center of the quantized enveloping algebra and “quantum differential operators”.

I also have an interest in the space consisting of “quantum differential operators”. Can this space be constructed in general? In the classical case, that is $q = 1$, differential operators are generated by $\frac{\partial}{\partial x_i}$ and x_j , where $\{x_i\}$ is a coordinate system. In the quantum case are there the generators corresponding to $\frac{\partial}{\partial x_i}$? I want to investigate the structure of the space of “quantum differential operators”.

Furthermore, I will construct “quantum differential operators” on a quantum analogue of a general algebraic variety with an action of an algebraic group except for G/P .