Research plan

I plan to study properties of knots through polynomial invariants from a view of knot diagrams with a consequence of previous researches. In particular, the aim of my research is mainly devoted to a study of properties of positive knots. My concrete research plan consists of four parts as follows.

1. Minimal diagram of positive knots

One of the central problems in Knot Theory is to determine whether a given diagram is minimal or not. It is, however, quite hard to approach to this problem in general. Jones revolution in the middle of 80's gave the complete answer of this problem to alternating knots. Namely it is proved that any reduced alternating diagram is minimal. For considering this fact in the case of positive knots, the problem is the following: *Is a minimal diagram of a positive knot realized by a positive diagram?* I obtained a partial affirmative answer for positive alternating knots in the paper [2]. Recently, Stoimenow found a positive knot without positive diagrams of minimal crossings. Hence, the problem for positive knots is negatively solved. On the other hand many positive knots have a positive diagram of minimal crossings, for example, torus knots, positive alternating knots, positive knots of genus one or two and so on. I observe the geometric and algebraic difference between positive knots with positive diagrams of minimal crossings and positive knots without positive diagrams of minimal crossings. At first I construct an infinite family of positive knots without positive diagram of minimal crossings, since such knots are not found so many, for example Stoimenow's knot and the positive knot constructed in the paper [6]. I show that the infinite family of positive knots proposed in the paper [6] is such a family. A method to prove the fact that a positive knot has no positive diagram of minimal crossings in the paper [6] is effective only on positive knots with the small crossing number. I improve such a method to be effective on positive knots with every crossing number. On the other hand, the positive knot without positive diagrams of minimal crossings constructed in [6] is a fibered knot of genus 3, that is, they have strong algebraic and geometric properties. So I also plan to construct positive knots with stronger properties.

2. The complements of positive knots

Tait conjectured that any pair of alternating diagrams presenting the same knot can be transformed to each other by a finite sequence of the local move on a diagram, called *flype* in the end of 19th century. This conjecture was proved by Menasco-Thistlethwaite. Their technique to prove this conjecture has been developed by a series of works of an analysis of the intersection of surfaces in the alternating knot complement. I plan to improve their technique to be able to apply it to the positive knot complement and to discover a new point of view.

In the process of the deformation from a positive diagram with the minimal crossings as a positive diagram into the minimal diagram (i.e., non-positive) for the positive knot constructed in [6], the number of crossings is increased once, and is decreased after that. I give a characterization of the process of the deformation of this type in general by observing the intersection of surfaces in the complement of a positive knot without positive diagrams of minimal crossings. In particular, I observe the intersection of a certain surface and a canonical Seifert surface from a positive diagram in the positive knot complement. For the study of the positive knot complement, Ozawa's characterizations of essential closed surfaces in the positive knot complement gives me a starting point. Canonical Seifert surfaces derived from positive diagrams decomposed into canonical Seifert surfaces derived from positive alternating diagrams by "Murasugi sum decomposition". So it is expected that relationships between positive knots and alternating knots in the study of surfaces in the knot complement.

As an application of this study, I characterize satellite positive knots. This is useful for decision of hyperbolicity of knots yielding Lens spaces ans small Seifert fiber spaces by Dehn surgeries. Hence this study connects to Exceptional surgery problem or Geometrization conjecture of 3-manifold.

3. Estimates for numerical geometric invariants by algebraic invariants

Many knot theorists found estimations for numerical geometric invariants by computable algebraic invariants, for example, for the genus by the degree of the Alexander polynomial, for the crossing number by the degree of the Jones polynomial, for the braid index by the degree of the HOMFLY polynomial, for the unknotting number by the knot signature and so on. I plan to find new knot invariants and new estimations for geometric invariants.

Recently, new knot invariants are defined by using an algebraic system, called Quandle. Those invariants are applied to 2-knot theory. An invariant derived from Quandle gives a lower bound for the minimal number of triple points in a projection of a 2-knot. An invariant of this type is constructed from Quandle, an abelian group and its representation. I construct another invariant derived from Quandle, which can be applied to find new estimations for numerical geometric invariants of a 1-dimensional knot. In particular, I give a skein relation to a invariant of this type to be able to calculate from a (positive) diagram directly.

By recent progress of the study of Knot floer homology, it is known that Knot floer homology estimates the genus and 4-genus of knots after the huge amount of the calculation. I also plan to find a way to reduce the amount of the calculation of Knot floer homology.

4. On canonical Seifert surfaces of knots

A canonical Seifert surface for a knot is obtained from a diagram by applying Seifert's algorithm. This algorithm is a good one from a viewpoint of the construction of Seifert surfaces for knots. However, a question *what is a geometrical interpretation of this algorithm?* should be proposed. So it is important that we give a geometrical characterization of a canonical Seifert surface.

I plan to solve the conjecture that the crossing number of a knot coincides with the canonical genus of its Whitehead double. I believe that I can show that this conjecture is true for alternating knots by an improvement of the technique in the papers [4] and [7].

At first, I improve a technique in the papers [4] and [7]. Moreover, I have known that I cannot show that this conjecture is true for the other knots by the technique in the papers [4] and [7]. I plan to develop a new technique to this conjecture.

In the paper [8], I showed that any canonical Seifert surface is ambient isotopic to a braidzel surface. I plan to construct a normal form of canonical Seifert surfaces through a study of braidzel surfaces.

Reference

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