Summary of Research

My research area is Knot Theory and its related topics. I am mainly interested in numerical geometric invariants, polynomial invariants of knots and their relationship from a view of knot diagrams. My typical results are the following:

1. Properties of positive knots

Knot Theory is a study of a "position" of a knot in a manifold. Fundamental problems is to give a "good position" to each knot type and to present it by a diagram.

In a knot diagram, each crossing has a sign, positive or negative in general. A diagram is said to be *positive* if it has only positive crossings. A knot with a positive diagram is called a *positive knot*. My research interest for positive knots is under the slogan "What geometric property does appear by the property such that all crossings are positive through polynomial invariants?"

In the paper [1], I showed that the positive knots with the unknotting number one are twist knots. *The unknotting number* of a knot is the minimal number of crossing changes needed to create a diagram of the trivial knot. Twist knots have obviously the unknotting number one. So it can be said that positive knots are complicated in general. In order to obtain this fact, I showed that the positive knots are contained in Rudolph's quasipositive knots, which appear at the intersection of a certain algebraic curve and the unit sphere in the complex 2-plane. Then I obtained that the 4-genus of a positive knot is calculated from a positive diagram.

A diagram of a knot is said to be *minimal* if the number of crossings in it is minimal among all diagrams presenting the knot. In the end of 19th century, Tait conjectured that a reduced alternating diagram is minimal, and proved by Kauffman, Murasugi and Thistlethwaite independently in 1987. For considering this famous fact in the case of positive knots, I observed the knots with an alternating diagram and a positive diagram respectively (which are called *positive alternating knots*) in the paper [2]. Then I obtained that a reduced alternating diagram of a positive alternating knots is positive. Hence a minimal diagram of a positive alternating knot is realized by a positive diagram.

Recently, Stoimenow found a positive knot without positive diagrams of minimal crossings. I observed this knot precisely, and found a tangle giving the above property essentially to Stoimenow's knot. I constructed another positive knot without positive diagrams of minimal crossings by using this tangle, and proposed a candidate for an infinite family of such positive knots in the paper [6].

The fact that any knot can be presented by a closed braid is first proved by Alexander. The *braid index* of a knot is the minimal number of strings among all closed braid presentations of the knot. Morton, Franks-Williams give a lower bound of the braid index called the *MFW bound* in terms of the reduced degree of the HOMFLY polynomial. By this bound, the braid indices of 2-bridge knots and torus knots are completely determined. Franks-Williams conjectured that the braid index of a closed positive braid (which is the closure of a braid presented by only positive powers of Artin's braid generators) coincides with the MFW bound. However, Morton-Short found a counter-example for this conjecture. In the paper [5], I showed that the braid index of a closed positive braid is two if and only if the MFW bound is two. I also give an infinite family of closed positive braids such that the difference between the braid index and the MFW bound is one. For these results, I observed precisely resolution trees to compute the HOMFLY polynomial, and defined a special type of it said to be a positive resolution tree. The possibility of the existence of a positive resolution tree for knots played an important role in the paper [6].

2. Canonical genus of knots

A knot spans a compact, connected, orientable surface in the 3-sphere, called a Seifert surface. The minimal genus among all Seifert surfaces of a knot is called the *genus* of the knot. A Seifert surface is said to be *canonical* if it is constructed from a diagram by applying Seifert's algorithm. The minimal genus among all canonical Seifert surfaces of a knot is called the *canonical genus* of the knot. In general, the canonical genus is greater than or equal to the genus of a knot. Many researchers constructed knots with the large difference between the genus and the canonical genus. These knots are satellite knots or composite knots. I constructed an infinite family of hyperbolic fibered knots with the arbitrarily large difference between the genus and the canonical genus in the paper [4].

It is conjectured that the crossing number of a knot coincides with the canonical genus of its Whitehead double. In the paper [7], I showed that this conjecture is true for 2-bridge knots by an application of a technique in the paper [4].

A spanning surface for a link is said to be a pretzel surface if it consists of two disks joined by several half-twisted, unknotted vertical bands. We can regard that the cores of the bands of a pretzel surface form the n-string trivial braid. From such a point of view, we can generalize pretzel surfaces to *braidzel surfaces*. A spanning surface for a link is said to be a braidzel surface if it consists of two disks joined by n bands with several half-twists so that the cores of the bands form an n-string braid. In the paper [8], I showed that any canonical Seifert surface is ambient isotopic to a braidzel surface. Then I define the *braidzel genus* for a link, which is the minimal genus among all braidzel surfaces for the link. In general, the braidzel genus is less than or equal to the canonical genus. I constructed an infinite family of links with the braidzel genus which is less than the canonical genus.

3. On double-torus knots

One of the generalizations of the connected-sum of knots is the *band-sum*. I showed that the Alexander polynomial of a band-sum of two knots is the product of those of summands and a certain ribbon knot by using a Seifert matrix in the paper [3].

A knot is said to be a *double-torus knot* if it can be embedded in a Heegaard surface in the 3-sphere. Hill-Murasugi showed that the Alexander polynomial of a certain double-torus knot contains that of a 2-bridge knot as a factor by algebraic calculations. In the paper [3], as an application of the above result of the Alexander polynomial of the band-sum, I gave a geometrical view to Hill-Murasugi's theorem such that the Alexander polynomial of a certain double-torus knot has a 2-bridge knot factor by deforming such a double-torus knot into the connected-sum of a 2-bridge knot and its mirror image through local moves preserving the Alexander polynomial. I also gave a generalization of Hill-Murasugi's theorem. These results and local moves in the paper [3] are applied to the study of fiberedness of double-torus knots by Hirasawa-Murasugi.

Reference

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