The history of the mathematical physics tells us that new physical aspects of a system or more fundamental perspectives can be recognized in revealing the hidden symmetry of a system and reformulating the theory describing the system in such a way that the symmetry is realized manifestly and geometrically. Since the discovery of the Jones polynomial, knot theory has provided a key for this in various fields such as conformal field theory, quantum group theory, low dimensional topology, solvable lattice model theory. Based on my results of the geometrical and algebraic studies of superstring theories and M-theory, I hope to elucidate the geometrical meaning of quantum invariants from the viewpoint of the mathematical physics.

Witten formulated a topological invariant (Witten invariant) of a three-manifold as a Feynmann path-integral of a WZW model. This is rigorously reformulated by Reshetikhin and Turaev based on the Dehn surgery and the Kirby calculus and by Turaev and Viro based on the triangulation and the quantum 6j-symbol. However, it seems that the geometrical meaning of the Witten invariant has not been fully clarified. I hope to clarify this point in my study as follows.

(a) The Witten invariant can be formulated for a three-manifold with boundaries and links using the topological field theory based on Atiyah's axiom. I want to examine how geometrical information such as boundaries and links are encoded in the invariant. It may be useful to use the invariant constructed using the S-matrix of the modular transformation in a conformal field theory, and to compare the invariant of the three-manifold *with* boundaries and links with those of the three-manifold *without* boundaries and links. For this purpose, I intend to examine and construct monodromy representations of the Knizhnik-Zamolodchikov equation for WZW models on supergroups and quotient groups, with the help of the studies in the Liouville theory and the H_3^+ WZW model. Further, I want to extend this study to the case with boundaries and links. Using the method employed in [23,25,29], possible boundaries of an open supermembrane (three-manifold with boundaries) can be classified. It is interesting to compare the result with that obtained from conformal field theory analysis.

In order to examine how geometrical information is encoded in the invariant, it is expected to be useful to work in the formulation developed by Prof. Kohno based on the Heegaard decomposition of a three-manifold and mapping class groups. As the classification of Riemann surfaces has played an important role in the formulation of string field theories, the Heegaard decomposition is expected to be important to formulate membrane field theories.

(b) Supersymmetric topological field theories define a generalization of the Witten invariant. In particular, I would like to examine the supersymmetric SL(2)/U(1) WZW model because this is expected to be dual to the N=2 Liouville theory from the AdS/CFT correspondence. It is interesting to compare invariants obtained in the both sides. In addition, the SL(2)/U(1) WZW model is expected to be dual to sin-Liouville theory from AdS/CFT correspondence, and I would like to examine this model also.

(c) Green-Schwarz string theory on the plane-wave geometry was shown to be exactly solvable in spite of the presence of mass terms. A modular invariant partition function was constructed in terms of massive theta-functions. Using the S-matrix of this modular transformation, invariants are expected to be constructed. Because the plane-wave does not have any cycle but is equipped with a gauge flux, it is interesting to examine the effect of the flux in the invariants. I expect that this leads to a hint to understand the geometrical meaning of the massive theta-function.

(d) It is known that the coefficient of the perturbative expansion of the Witten invariant can be determined using the Chern-Simons perturbation theory and lead to the Vasiliev invariants. For (a)-(c), it is interesting to examine the Chern-Simons perturbation theory and Vasiliev invariants.

I expect that these studies may lead to the rigorous definition of the Feynmann path-integral and the method to extract combinatorial invariants out of the Feynmann path-integral.