Summary of my research

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Papers are referred by numbers in “List of may papers”

Due to the Hyperbolic Uniformization Theorem for Haken manifolds by Thurston the complements of almost all knots and links admit complete hyperbolic structures of finite volume. (We call such a manifold a cusped hyperbolic manifold.) Epstein and Penner defined an ideal polyhedral decomposition of a cusped hyperbolic manifold which is determined canonically from the hyperbolic structure. The decomposition is called the canonical decomposition of a manifold. I study to reveal the relationship between the hyperbolic and the topological structures of a cusped hyperbolic 3-manifold by means of canonical decomposition. My study so far is classified roughly into three lines; the first is a research on the relationship between Dehn surgeries and canonical decompositions, the second is the one between Heegaard splittings and hyperbolic structures and the third is the study of infinite volume hyperbolic manifolds from the view point of combinatorial structures.

Dehn surgeries and canonical decompositions: I have studied the behavior of the canonical decompositions of manifolds in the infinite family of cusped hyperbolic 3-manifolds obtained from a hyperbolic 3-manifold with at least 2 cusps by Dehn surgeries ((1), (2), (3), (11), (13), (18)). As a result, I could prove that the canonical decomposition of almost every manifold obtained in such a way is the union of the “stable part”, which is hardly affected by the way of filling the end by a solid torus, and the “unstable part”, which is determined by the element of the mapping class group of the torus characterizing the way of filling. (See (18) for an outline; details is in preparation.) For a hyperbolic 3-manifold with at least 2 cusps, the “stable part” can be determined by using the decomposition of Epstein-Penner’s type with weight 0 on the cusps which will be filled by the surgery, and weight 1 on the remaining cusps. If we further assume that the combinatorial structures on the cusps which remain unfilled are “generic”, then the canonical decomposition of almost every manifolds obtained by a Dehn surgery is the union of a subdivision of the “stable part” and the “unstable part” which is combinatorially isomorphic to the solid torus realized as a layer of ideal tetrahedra.

Heegaard splittings and canonical decompositions: With Makoto Sakuma, Masaaki Wada and Yasushi Yamashita I have determined the canonical decomposition of every hyperbolic two-bridge knot (or link) complement by applying the Jorgensen’s theory on the deformation of Ford domains of quasifuchsian punctured torus groups to the four times punctured sphere, and then extending it to that for deformations of cone-manifolds. (See (4), (14), (17) for an outline; details is in preparation.) The canonical decomposition determined in such a way is seen to be coincide with the topological ideal tetrahedral decompositions given by Sakuma and Weeks as candidates to the canonical decompositions. In our method, a continuous family of hyperbolic structures, each of which has cone singularities along the unknotting tunnels of the two-bridge knot under consideration called the upper and the lower tunnels, is constructed. The study in this line can also be regarded as the first step to the construction of “hyperbolic Heegaard splitting theory”.

Hyperbolic manifolds of infinite volume: Jorgensen’s theory is seen to be able to be applied to the study of the Ford domains of geometrically infinite boundary groups. As a result, I could determine the canonical decomposition of any hyperbolic once-punctured torus bundle. (See (15) for an outline; details is in preparation.) Lackenby have independently determined the canonical decompositions of once-punctured torus bundles by using a combinatorial argument of polyhedral decompositions. This proves a beautiful correspondence of hyperbolic geometry and topology with canonical decompositions in between.

Recall that we have used deformations of infinite volume hyperbolic manifolds in order to construct two-bridge knot complements. Actually, it is possible to determine the canonical decompositions of infinite volume cusped hyperbolic manifolds, which contain more informations than the combinatorial structures of Ford domains (see (7)). The quasifuchsian space of punctured torus is conjectured to be decomposed in a nice way, by using the canonical decompositions of infinite volume manifolds, and in fact, there are some numerical supporting evidences obtained by using a computer (see (8)).

McShane defined an infinite series, the McShane series, which is constant on the Teichmüller space of a hyperbolic surface, and Bowditch proved that the McShane series is constant on the quasifuchsian space of the once-punctured torus. Bowditch also proved that the value of a certain sub-series is equal to the modulus of cusp of a once-punctured torus bundle over the circle. With Makoto Sakuma and Hideki Miyachi (9), I proved that the McShane series converges absolutely at any geometrically finite boundary of the quasifuchsian space of the once-punctured torus. Moreover, we proved that the imaginary part of the value of a certain sub-series is equal to the “width” of the limit set. The results on the quasifuchsian space and for the bundles over the circle to once-punctured torus can be generalized to any punctured surface of finite type (10).