

My research plan

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Virtual knots are generalization of knots. They correspond to abstract knots. They are useful to study invariants of links on surfaces systematically. They are also useful to study quandle homology groups. I am going to study on virtual knots.

It is the main theme of knot theory to make a classification table of knots (called the knot table). A lot of knot theorists have been trying to make up the knot table. T. Kishino made a virtual knot table whose crossing numbers are six or less. There is a one-to-one correspondence between virtual knot diagrams and Gauss codes. From this point of view, virtual crossings are meaningless, and it seems to be natural to classify virtual knots by the number of their real crossings without counting virtual crossings. I try to classify virtual knots by the number of real crossings. For this purpose, I will use computer to make a table of Gauss code to present virtual knots, and calculate some invariants.

It is important to characterize invariants for classification of knots. A lot of invariants of classical knots have been researched to characterize and used for classification of classical knots. Although virtual knots are generalization of classical knots, their invariants act generally differently from classical ones. So we need an approach, which is distinct from the classical case. For example, the Jones polynomials of virtual knots are quite different from those of classical knots. However, as shown in [4], when we consider virtual knots with checkerboard colorable diagram, the Jones polynomials of such virtual knots are the same with those of classical knots. I will study Jones polynomials and JKSS invariants for both virtual and classical knots to make the difference clear, and characterize these invariants for virtual knots. It is unknown whether there is a nontrivial knot with a trivial Jones polynomial. There are a lot of nontrivial virtual knots with trivial Jones polynomials. In the process of making virtual knot table and studying characterization of invariants, I also try to research on this problem.

A skein relation of a knot invariant means a relation of the invariant for a certain kind of triple of link diagrams called a skein triple. Skein relations were found for several invariants. They are useful not only to calculate invariants but also to characterize them. In [5] and [7], I showed new skein relations of Jones polynomials of virtual knots. I am also going to study about resolution trees with new skein relations. In [8] and Preprint [2], I showed the feature and some skein relations for the virtual knot invariant which Y. Miyazawa defined. In October of 2004, he introduced a method to construct new invariants systematically, including his original invariants. I am going to study about it.

Virtual knot diagrams can be considered as link diagrams on a surface. From this viewpoint, T. Kadokami has been investigating projective virtual knot diagrams, which are obtained from virtual knot diagrams by replacing real crossings with vertices. The minimum genus of a closed oriented surface where a virtual knot can be embedded is an invariant of the virtual knot. I am going to study about it.