Research Plan

As I explained in the report on my works, I have been studying complex surfaces of general type with small geometric genus, in view of the fundamental groups and the torsion parts of the Picard groups. I plan to continue the studies in the same direction for a few more years, while at the same time broadening the research area. In what follows, every varieties are defined over the complex number field $\mathbb{C}$. For a regular surface X, we denote by c_1^2 and χ, the first Chern number and the Euler characteristic of the structure sheaf, respectively. We denote by $\mathrm{Tors}(X)$ the torsion part of the Picard group, and call it the torsion group for short.

First, as for the continuation of my recent works (i.e., studies on surfaces of general type in view of the torsion groups), I plan to study the cases of small $c_1^2 - 2\chi$ first. Note that lines $c_1^2 - 2\chi =$ constant are parallel to the Noether line. In addition to the problems mentioned in the report on my works, I intend to study Torelli type problems for these surfaces intensively for the next few years. The following three for the case $c_1^2 = 2\chi - 1$, among these problems, are my next goals:

- the weak global torelli problem for the case $\chi = 2$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/3$,
- concrete descriptions for the surfaces of the case $2 \leq \chi \leq 3$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/2$,
- Torelli type problems for the case $2 \leq \chi \leq 4$ and $\mathrm{Tors}(X) \simeq \mathbb{Z}/2$.

Surfaces of general type with small geometric genus are interesting objects, which tend to take several topological types under the same same values of numerical invariants. Meanwhile, these objects are quite difficult to study, since they often appear as exceptions for the general theorems or for general treatment. For instance, absence of the canonical map makes it extremely difficult to study surfaces of general type with geometric genus $p_g = 0$ or 1. These objects are interesting also from the view point of the fundamental groups. Whether there exist surfaces which are algebraically simply connected and not simply connected still remains as a challenging problems. By G. Xiao' s theorem, any minimal regular surfaces satisfying $c_1^2 < (8/3)(\chi - 2)$ and $c_1^2 < 3p_g - 13$ are simply connected. Thus when we study surfaces on a line parallel to the Noether line, some of these surfaces with small geometric genus are good candidates for algebraically simply connected, but non-simply-connected surfaces. And also, I am expecting that these surfaces of small geometric genus might give good examples for a method to attack global Torelli theorem via degenerations, which K. Kato and S. Usui are developing.

Next, as for other topics, I intend to study surfaces of general type whose canonical maps are composed of pencils. I am in particular interested in describing surfaces of boundary cases. Several results are known on surfaces with canonical pencils: A. Beauville proved, in one of his early works, a certain inequality for numerical invariants of such surfaces, and gave a bound for the genus of the general fibers of the Stein factorizations; G. Xiao gave a bound for the irregularities, etc. Many of the known inequalities however are not best possible, and many problems are left. There are some other other topics. R. Pardini recently proved what is known as Severi's inequality: if the Albanese image of a minimal complex surfaces of general type has maximal dimension($= 2$), then $c_1^2 \geq 4\chi$ holds. I am interested in the problem of describing surfaces near the boundary, for which Manetti's result, for instance, are known.

I am also interested in some other topics, e.g., local invariants of families of curves, but I stop here this report. I regard the next few years as a period for broadening my research area.