My work

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In 1988, E. Witten proposed a topological invariant of a 3-manifold, the quantum G-invariant, by using a compact simple Lie group G. N. Yu. Reshetikhin, V. G. Turaev, R. Kirby and, P. Melvin gave a mathematical proof for existence of the invariant and found a relationship between the quantum $PSU(2)$ invariant of a 3-manifold obtained from S^3 by surgery along a link and its Jones polynomial. H. Murakami showed a relationship between the quantum $PSU(2)$ invariant and the Casson-Walker invariant of a rational homology 3-sphere. In [1], I gave a similar relationship between the quantum $PSU(2)$ invariant and the Casson-Walker invariant of a 3-manifold whose $Z/5Z$ Betti number is positive.

In 1985, V. F. R. Jones defined the Jones polynomial, via a certain von Neumann algebra, which is one of the most important discoveries in knot theory. On the other hand, L. H. Kauffman defined a regular isotopy invariant of a link diagram on S^2, called the Kauffman bracket polynomial, by using a state model. He proved that his invariant, after normalization with the writhe effect, is equal to the Jones polynomial. In [2], I generalized the Kauffman invariant for a link diagram on S^2 to that for a link diagram on any closed oriented connected surface F, and generalized the Jones invariant for a link in S^3 (R^3 or $R^2 \times R$) to that for a link in the product space $F \times R$ of F and the real line R. The Kauffman-Murasugi-Thistlethwaite theorem—Any proper alternating connected link diagrams of a link in S^3 have the same number of double points—was generalized for a link in the thickened surface $F \times R$.

In the late 1990's L. H. Kauffman introduced virtual knot theory. It is a generalization of knot theory, which was motivated by the study of knot diagrams in a closed oriented surface and abstract Gauss codes. A virtual knot diagram is obtained from a knot diagram by replacing some crossings with virtual crossings. He generalized the Jones polynomials of knots to those of virtual knots. M. Goussarov, M. Polyak, and O. Viro studied the Vassiliev invariant by using virtual knots. D. S. Silver and S. G. Williams characterized the knot groups of virtual knots. An abstract link diagram is a pair of a compact oriented surface and a link diagram on it such that the diagram is a deformation retract of the surface. In [3], S. Kamada and I showed that there is a natural bijection between virtual knots (links) and abstract links. Using abstract links, we gave a geometric interpretation of virtual knot groups. Jones polynomials of virtual knots have quite different feature from those of classical knots. In [4], it is proved that Jones polynomials of virtual knots that are presented by checkerboard colorable diagrams have a certain property that classical knots have. A skein relation among the Jones polynomials of virtual link diagrams is given in [5]. In [7], two kinds of skein relations of the Jones polynomials of virtual link diagrams are given, which is complementary to the previous one. In [6], I generalized the relationship between the Jones polynomials and the numbers of crossings of v-alternating links. In 2004, Y. Miyazawa introduced an invariant for a virtual link and conjectured that this invariant splits the Jones polynomials in a certain sense. In [8], I gave an affirmative answer to his conjecture. In my preprint [2], three kinds of skein relations of Miyazawa's invariants of virtual link diagrams are given.

In the preprint [3], A. Ishii, S. Kamada and I studied a skein module, called the virtual magnetic skein module, which is due to Jones polynomials for virtual knots, and determined the rank of this module. Furthermore we proved that the skein relations are derived from it In the preprint [4], a 2-variable invariant for a long virtual knot due to Jones polynomial and Miyazawa polynomial is introduced. The invariant satisfies a product formula.

In the preprint [5], I constructed a table of virtual knots whose numbers of real crossings are equal or less than 4, equipped with their Jones polynomials, Miyazawa polynomials and JKSS invariants. Furthermore, I determined the virtual crossing numbers of more than half virtual knots in the table.