My research plan.

By joint work with H. Yoshida, we conjectured a formula for the p-adic period and a new invariant $X_p^\sigma(\mathfrak{c})$. This invariant is defined by factorizing the value of the derivative of the p-adic partial ζ-function at $s=0$ and is expressed in terms of p-adic multiple Γ-functions. Our aim is to generalize and prove our conjecture. Moreover we want to apply our conjecture to some problems. We will explain some ideas.

1. On (p-adic) multiple Γ-functions. Algebraic relations among special values of multiple Γ-functions (e.g., the Deligne-Koblitz-Ogus product relations on classical Γ values) are interesting. We can regard these relations as being related to arithmetic geometry. For example, by using Yoshida's conjecture, we can get many relational expressions. Moreover there exist similar relations among special values of p-adic multiple Γ-functions. Note that the number of relational expressions increases in the p-adic case. We can see a similar result on the dimension formula for the spaces of (p-adic) multiple zeta values. We should be able to explain these phenomena using arithmetic geometry.

2. On the p-adic period. Although there exist many research results on the p-adic period (e.g., by N. Katz, G. Faltings, A. Ogus, etc.), R. F. Coleman's calculation on Fermat curves may be the only result about its specific values. In this sense, our research is very important since we formulated the relational expression between the p-adic period and the special values of p-adic multiple Γ-functions. Furthermore we want to describe specific values of p-adic periods strictly.

3. On our conjecture about p-adic periods and $X_p^\sigma(\mathfrak{c})$. We formulated our conjecture strictly in the p-ordinary case. Furthermore in any case, we conjecture a formula:

$$\log_p\left(p_{p,K}(\mathrm{id},\tau)^{1-\varphi_{\mathrm{cris}}^{f_{\mathfrak{p}}}}\right) \equiv -\frac{\mu(\tau)}{2}\log_p(\mathfrak{p}) + \frac{1}{|G|}\sum_{\chi\in\hat{G}_-}\frac{\chi(\tau)\sum_{\mathfrak{c}\in C_{\mathfrak{f}_\chi\mathfrak{p}}}\chi(\mathfrak{c})X_p^{\mathrm{id}}(\mathfrak{c})}{L(0,\chi)}. \tag{1}$$

In fact, we can show such a formula when $F=\mathbf{Q}$. We want to show this conjecture and Yoshida's original conjecture. To prove them fundamentally, we have to calculate (p-adic) periods but it seems very difficult. Therefore we will try some indirect demonstrations. That means, if we assume these conjectures to be true then the right hand side of these formulas should have the same properties as those of (p-adic) periods. In particular, we notice how these values change if we extend or embed the fields F, K. We can formulate such variations by arithmetic geometry so we will verify them.

Alternatively, we may prove our conjecture by an algebraic method in some cases. Our conjecture is consistent with the Gross-Koblitz formula if K is a quadratic imaginary field and p splits in $K/\mathbf{Q}$. Although the original proof of this formula involved the use of cohomological methods, A. M. Robert showed it by using a power series expansion of the p-adic Γ-function. We will study on the p-adic multiple Γ-function and apply this method to our conjecture.

4. On the construction of class fields. Our conjecture deeply involves Gross conjecture, which is a p-adic analogue of Stark conjecture. A direct consequence of such conjectures is the construction of class fields. In fact assuming our conjecture, we can get the Brumer-Stark element and can construct any class field which is a CM-field over any totally real field. Moreover, we may apply our method to prove the Brumer-Stark conjecture and Gross' conjecture.

5. On the higher derivative of the (p-adic) L-function. Yoshida considered the higher derivative of the L-function and showed some formulas. I formulated the second derivative of the p-adic L-function. We can see a certain similarity between them as in the case of Shintani's formula and its p-adic analogue. We want to prove the same result on any higher derivatives. We defined new invariants by factorizing the first derivatives and we can get the relational expression of (p-adic) periods and can prove Gross' conjecture partially. Therefore by factorizing the higher derivatives, we

will get new invariants, which should be important in number theory and be useful for proving Gross' conjecture.

6. On the other p-adic L-functions and their p-adic periods. We want to generalize our result to other fields. For example, if we assume $F = \mathbf{Q}$ and the p-ordinarity then our conjecture is consistent with the theory of L-invariants of p-adic L-functions with a trivial zero. It is studied by R. Greenberg and H. Hida. In the general case, we can regard our invariants as being subdivisions of their L-invariants. Therefore our conjecture should be generalized to the p-adic L-function of the modular form and its p-adic period. Especially, we want to generalize to the case without the p-ordinarity. Moreover we are interested in the Iwasawa conjecture and the equivariant Tamagawa number conjecture since these seem close to our research field.