My research abstract.

Let F be a totally real algebraic number field, $\mathfrak{c}$ an ideal class of a suitable ideal class group, $\zeta_F(s,\mathfrak{c})$ the partial ζ-function of $\mathfrak{c}$. By using Shintani's formula and H. Yoshida's lemmas, we get a canonical factorization:

$$\zeta_F'(0,\mathfrak{c}) = \sum_{\sigma\in J_F} X^\sigma(\mathfrak{c}),\ X^\sigma(\mathfrak{c}) := \sum_{j\in J}\sum_{z\in R(\mathfrak{c},j)} L\Gamma_{r(j)}(z^\sigma, v_j^\sigma) + \sum_{i\in I} a_i^\sigma \log(b_i^\sigma). \tag{1}$$

Here we denote by J_F the set of all isomorphisms of F into $\mathbf{C}$ (or $\mathbf{C}_p$), by v_j a certain $r(j)$-row vector whose components are totally positive integers in F, by $R(\mathfrak{c},j)$ a certain finite subset of F, by a_i, b_i certain elements of F. $L\Gamma_r(z,v) := \log(\frac{\Gamma_r(z,v)}{\rho_r(v)})$ is Barnes' multiple Γ function, which is a generalization of the classical Γ-function. Yoshida conjectured for $\tau \in G := \mathrm{Gal}(K/F)$

$$p_K(\mathrm{id},\tau) \equiv \pi^{-\mu(\tau)/2}\exp\left(\frac{1}{|G|}\sum_{\chi\in\hat{G}_-}\frac{\chi(\tau)\sum_{\mathfrak{c}\in C_{\mathfrak{f}_\chi}}\chi(\mathfrak{c})X^{\mathrm{id}}(\mathfrak{c})}{L(0,\chi)}\right) \mod \overline{\mathbf{Q}}^\times. \tag{2}$$

We can regard this formula as a generalization of the Chowla-Selberg formula. Here K is a CM-field and we assume that K/F is an abelian extension. We denote by $\hat{G}_-$ the set of all odd characters of G, by $\mathfrak{f}_\chi$ the conductor of χ, by $C_{\mathfrak{f}_\chi}$ its ideal class group. For $\tau = \mathrm{id}, \rho$ (complex conjugation) and otherwise, we put $\mu(\tau) := 1, -1, 0$ respectively. p_K is Shimura's CM-period symbol, which is defined by factorizing the values of geometric periods of Abelian varieties with complex multiplication by K. We call the right hand side of (1) the absolute CM-period. Note that the invariant $X^\sigma(\mathfrak{c})$ involves the Stark-Shintani conjecture deeply. The background of these researches is the importance of the leading term in the Taylor expansion of the L-function, e.g. the class number formula.

My results are as follows. First, we define the p-adic multiple Γ-function $L\Gamma_{p,r}$ by the derivative of the p-adic multiple ζ-function at $s=0$. Then we get $L\Gamma_{p,1}(z,(1)) = \log_p(\Gamma_p(z))$ with Morita's p-adic Γ-function Γ_p. (cf. $L\Gamma_1(z,(1)) = \log(\Gamma(z)) - \frac{1}{2}\log(2\pi)$.) Furthermore we get a p-adic analogue of Shintani's formula:

$$\zeta_{p,F}'(0,\mathfrak{c}) = \sum_{\sigma\in J_F} X_p^\sigma(\mathfrak{c}),\ X_p^\sigma(\mathfrak{c}) := \sum_{j\in J}\sum_{z\in R(\mathfrak{c},j)} L\Gamma_{p,r(j)}(z^\sigma, v_j^\sigma) + \sum_{i\in I} a_i^\sigma \log_p(b_i^\sigma), \tag{3}$$

with the same notation as in (1). Here $\zeta_{p,F}$ is the p-adic partial ζ-function. This formula gives a generalization of the Ferrero-Greenberg formula. As a corollary, we can show that

$$\text{if } r(\chi) := \{\mathfrak{p}|(p),\ \chi(\mathfrak{p}) = 1\} \geq 2 \text{ then } \mathrm{ord}_{s=0} L_p(s,\chi\omega) \geq 2. \tag{4}$$

Here ω is a character which is the composite mapping of the Teichmüller character and the ideal norm map. This is a partial solution of Gross' conjecture, which states a relational expression between the leading term in the Taylor expansion of the p-adic L-function at $s=0$ and the p-adic regulator.

Using this formula, Yoshida and I formulated a p-adic analogue of Yoshida's original conjecture (2). Let $p_{p,K}$ be the p-adic period symbol, witch is a p-adic analogue of the CM-period symbol. $p_{p,K}$ takes the values in a certain algebra B_{cris}. Then we can show for $\tau \in J_K$

$$\log_p\left(p_{p,K}(\mathrm{id},\tau)^{1-\varphi_{\mathrm{cris}}^{f_{\mathfrak{P}}}}\right) = \frac{1}{2}\log_p\left(\mathfrak{P}^{(\rho-\mathrm{id})\tau^{-1}}\right). \tag{5}$$

Here we take a prime ideal $\mathfrak{p}$ (resp. $\mathfrak{P}$) which induces the p-adic topology on F (resp. K), $f_{\mathfrak{P}}$ is the degree of the prime ideal $\mathfrak{P}$ and φ_{cris} is the Frobenius action on B_{cris}. We define $\log_p(\mathfrak{a}) := \frac{1}{h}\log_p(\alpha)$

if an integral ideal $\mathfrak{a}$ satisfies $\mathfrak{a}^h = (\alpha)$. The first version of our conjecture is as follows. Assume that $\mathfrak{p}$ splits completely in K/F. Then for $\tau \in G$

$$(6) \quad \frac{1}{2}\log_p\left(\mathfrak{P}^{(\rho-\mathrm{id})\tau^{-1}}\right) \equiv -\frac{\mu(\tau)}{2}\log_p(\mathfrak{p}) + \frac{1}{|G|}\sum_{\chi\in\hat{G}_-}\frac{\chi(\tau)\sum_{\mathfrak{c}\in C_{\mathfrak{f}_\chi\mathfrak{p}}}\chi(\mathfrak{c})X_p^{\mathrm{id}}(\mathfrak{c})}{L(0,\chi)} \mod \mathbf{Q}\log_p(O_F^\times).$$

This formula gives a generalization of the Gross-Koblitz formula. We call the right hand side of (6) the p-adic absolute CM-period. By (5) and (6), we get the relational expression between the p-adic period and the p-adic absolute CM-period, which is a p-adic analogue of Yoshida's original conjecture (2). Note that we formulated a more precise conjecture. We can show that if we assume that our conjecture is true then Gross' conjecture in the case of $r(\chi) = 1$ is true. Moreover our conjecture is a factorization of Gross' conjecture.