Plan of my research for the future

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In a study of low-dimensional topology, it is very important to characterize geometric properties of manifolds by using algebraic methods. In my future research, I set two goals as follows.

Goal 1: Studing geometric structures of 4-manifolds via Khovanov theory

In a paper published in 1984, V. F. R. Jones gave an important polynomial invariant of a knot, now called the Jones polynomial of a knot using the von Neumann algebra. On the other hand, L. H. Kauffman constructed an invariant which is equivalent to the Jones polynomial by studing a diagram of a knot or a link combinatorially. Then in a paper of M. Khovanov published in 2000, the construction of Kauffman has been generalized and a cohomology such that the Euler characteristic is the Jones polynomial was constructed from a diagram of a knot or a link via (1+1)-TQFT. E. S. Lee modified Khovanov's TQFT and gave a useful cohomology. Moreover, J. Rasmussen succeeded in constructing an effective cobordism invariant of a knot from Lee's cohomology. In fact, Rasmussen has computed the unknotting number of a torus knot by using the invariant, that is he proved Milnor conjecture combinatorially. One of my goals is to study geometric structure of 4-manifolds by using Rasmussen invariant. In fact, we can show the existence of an exotic structure of 4-dimensional space by using Rasmussen invariant. This fact shows that the existence of an exotic structure of a 4-manifold is shown topologically and it is surprising because the existence of such a structure was only shown via gauge theory so far. I have shown the existence of an exotic structure of a certain Casson handle using Rasmussen invariant. The exoticness of a Casson handle can be considered as non-sliceness of certain knots. (A slice knot is defined as a knot which bounds a disk in 4-dimensional space.) I expect Rasmussen invariant to show the exoticness of the other Casson handles or 4-manifolds. On the one hand, I want to characterize Rasmussen invariant comparing the other invariants of knots.

Goal 2: Finding geometric structures reflected in quantum invariants

The research related with the volume conjecture which has suggested the relation of the simplicial volume (Gromov invariant) of the complement of a knot and the colored Jones polynomial shows that a quntum invariant is deeply related to a geometric structure of a manifold. For example, the colored Jones polynomial of a figure 8 knot gives the hyperbolic volume of its complement as a certain limit. I want to investigate a relation between the Jones polynomial of a knot and a geometric structure of a 3-manifold. I have already obtained a simple formula for the colored Jones polynomial of a doubled knot. As a corollary, I have shown that if the volume conjecture for untwisted positive (or negative) doubles of knots is true, then every nontrivial knot has nontrivial N-colored Jones polynomial. Then I want to study doubled knots concerning the volume conjecture. On the other hand, A. Stoimenow has given a list of potential counterexamples of 12, 13 and 14 crossings for the conjecture. I want to research the knots.