Research Plan

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We have researched well via quandles in knot theory and surface knot theory. Using knot quandles and quandle cocycle invaraints, we have studied some properties of surface links and classical links, for example, triple point number, triple point cancelling number, non-invertibility of surface links, chirality of classical links and so on. I plan to study surface links and others via quandles.

In [7] of the list of publications, I studied the w-indices of surface links via quandle cocycle invariants. Let θ_p be a 3-cocycle of dihedral quandle of order p such that p is an odd prime integer. We denote the qunadle cocycle invariants of a surface link F associated with θ_p by $\Phi_p(F)$. In [7], it was proved that if $\Phi_3(F)$ is non-trivial, then the w-index of F is at least 6. This bound is best possible since the w-index of 2-twist spun trefoil T_2 is 6 and $\Phi_3(T_2)$ is non-trivial. Similary, I would like to determine such integer when quandle is dihedral quandle of order $p \neq 3$, tetrahedral qunadle and so on.

In present, we often use dihedral qunadle in our researches via quandles. Though T. Mochizuki gave 3-cocycles of some Alexander quandle, in general, it is so hard to construct 3-cocycles. As generalizations of dihedral quandles, we can consider Burnside keis (a kei is quandle with a certain property) and core quandles of Burnside groups. I would like to consider 3-cocycles of these quandles. These may be useful to study some properties of surface links.

I would like to define qudandle cocycle invariants of singular surface links and study them, A singular surface link is a surface link with some transverse double points. As similar moves to Reidemeister moves for classical links, Roseman moves for surface links are known, i.e., two diagrams of same surface links are related to each other by some Roseman moves. Such moves have not been known for singular surface links. Thus, it is not easy to define qunadle cocycle invariants through thier diagrams. To start our program, I would like to study quandle cocycle invariants of singular surface braids. In the I. Hasegwa's doctor thesis, he researched state-sum invariants of surface braids via chart descriptions. Quandle cocycle invariants also can be defined as state-sum invariants, and we can reconstruct quandle cocycle invariants of surface links. We can apply its construction for singular surface braids, so we can define quandle cocycle invariants of them. Here, we remark that it is not known whether these invarinats are for singular surface links. I would like to study properties of singular surface braids via qunadle cocycle invariants of them.

In [5] of the list of publications, I proved that crossing changes for singular surface braids are unknotting operations. Thus, we can define unknotting numbers and triple point cancelling numbers of them associated with crossing changes, I would like to research whether quandle cocycle invariants give lower bound of them as given in [3] of the list of publications,

A quandle has a binary operation with three axioms. Such moves are corresponding to Reidemeister moves I, II, III, so we can construct knot quandles for knot diagrams, Such moves for spatial graph are given as six (or five if each vertex is flat vertex) type moves. Here, we consider a algebraic system as quandles with six (or five) axioms corresponding to those moves, so I would like to define invariants for diagrams like as a knot quandle. I also would like to consider whether cocycle invariants can be define well.