Result of research

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I have studied surface links via qunadle cocycle invariants and surface braids. A surface link is an oriented closed surface embedded in the 4-dimensional Euclidean space $\mathbf{R}^4$ locally flatly. J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford and M. Saito introduced qunadle homology theory, and defined invariants of surface links, which are called quandle cocycle invariants, for each qunadle 3-cocyle. A qunadle homology theory is a modification of rack homology theory defined by R. Fenn, C. Rourke, B. Sanderson. The notation of surface braids was introduced by O.Viro. It is known Alexander's theorem and Markov's theorem in dimension 4, and hence, surface braids are closely related to surface links. The results of my reserch will be given in the following list. The reference number in the following is correspond to the number in list of publications.

1. triple point cancelling number. It is known that attaching a finite number of 1-handles to a surface link F is an unknotting operation and triple point cancelling operation of F by F. Hosokawa and A. Kawauchi. Thus, we can define the triple point cancelling number of F. In [3], we detact a method to bound lower of the triple point cancelling numbers of F via qundle cocycle invariants of F. It is also see that there is a surface link F such that the triple point cancelling number of F is n for any natural number n.

2. w-index. The w-indices of a surface link F is the minimal number of triple points of surface braids whose closures are ambiently isotopic to F in $\mathbf{R}^4$. I. Hasegawa proved that there is a S^2-link whose w-index is 6. In recently, we detact a method to bound lower of the w-indices of F via qundle cocycle invariants of F([7]). It is also see that for any natural number l and non-negative integer $g_1, \dots, g_l$, there is an l-component surface link $F = F_1 \cup \cdots \cup F_l$ such that $w(F) = 6$ and the genus of F_i is g_i for any $i \in \{1, \dots, l\}$, where each F_i is a connected component of F.

3. calculations of qunadle cocycle invariants of twist spun links. We can also define invariants of classical links for each quandle 3-cocycle. The quandle cocycle invariants of twist spin of a classical link L can be calculated by using the invariants of L for same qunadle 3-cocycle. In general, we need the informations of signs of all double points to calculate the invariants of L, and it is complicated. Let θ_p be a 3-cocycle of dihedral quandle of order p such that p is an odd prime integer. We detact to be able to calcualte the invariants of L for θ_p without informations of signs of all double points, and we calcualted quandle cocycle invariants associated with θ_p of twist spun 2-bridge knot([2]) .

In [4], we calculated quandle cocycle invariants of twist spun pretzel links. In this case, we need essentialy to calcualte in $\mathbf{Z}/p^2\mathbf{Z}$. This is the main content of my doctor thesis.

In [5], we calculated quandle cocycle invariants of twist spun torus links. This result is extension of S. Asami and S. Satoh's result for twist spun torus knots.

4. crossing change of singular surface braids. A singular surface braid is a surface braid with transverse double points. A crossing change is an operation for a singular surface braid S inserting a pair of positive and negative crossing points along a chord that is a straight segment connecting adjacent sheets of S. In [6], we proved that crossing changes are unknotting operation of singular surface braids. By C. A. Giller, it is known that similar local moves of surface links are unknotting operations. We also proved the Giller's theorem as a consequence of our result.

5. finite type invariants of singular surface braids. S. Kamada introduced finite type invariants of surface links in $\mathbf{R}^4$ associated with crossing cahnges and 1-handle surgeries. In [1], we consider finite type invariants associated with 1-handle surgeries of singualr surface braids. These invariants are controled by three finite type invariants, which are double point number, Euler cahracteristic and normal euler number. In [6], we also consider finite type invariants associated with crossing changes of singualr surface braids. These invariants are controled by double point number and for each component, sheet number, Euler cahracteristic and normal euler number.