Research Plan

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I study the knot cobordism group and a slice-ribbon knot as its identity element, and finite type invariants of 2-bridge knots. Here I list the problems I am working on.

Slice-ribbon knots and knot cobordism invariants

1. Make a complete “ribbon knot atlas”. This is one of the basic works of knot theory.
2. Find a “mutation-sensitive” cobordism invariant: I found that the candidates of the unidentified ribbon knots are all mutants or mutants-like. I think it seems a key for studying knot cobordism group and slice-ribbon knots to have a mutation-sensitive cobordism invariant. Recently we have two strong cobordism invariants: One is given by J. Rasmussen and the other is given by P. Ozsvath and Z. Szabo. Rasmussen gave a cobordism invariant by using the Khovanov homology which is a homology-based extension of the Jones polynomial. Ozsvath and Szabo gave a cobordism invariant by using the Floer homology which is a homology-based extension of the Alexander polynomial. Unfortunately, both cobordism invariants generated by these homologies are not “mutation-sensive”, i.e., invariant under knot mutation. However, there is a remarkable property of the Floer homology. The Floer homology itself is mutation-sensitive. For instance, the Floer homology for the Kinoshita-Terasaka knot and the Conway knot are different. (They did not investigate the property for other mutants.) It seems important to study what kind of information we lose when we generate the cobordism invariant from the Floer homology.

Finite type invariants of 2-bridge knots

1. Derive some arithmetic results for two formulas for the Casson knot invariant of 2-bridge knots mentioned in my research statement.
2. Many works on 2-bridge knots are done by using Conway's normal form. By using the map for the Gauss diagram of Schubert's normal form mentioned in my research statement, some problems relating to Schubert's normal form may become easier. In fact, studying finite type invariants of 2-bridge knots seems to be the case. So I study finite type invariants of 2-bridge knots by using the Gauss diagram of Schubert's normal form and I want to derive some arithmetic results by using two formulas for them; one is a formula by means of a continued fraction expansion and another is a formula without using a continued fraction expansion.
3. Give a geometric interpretation for results on finite type invariants of PSL(2, Z) mentioned in my research statement. Furthermore, I want to extend this work to general mapping class groups.