Research Statement

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Invariants of 2-bridge knots (Publications [1, 2, 3, 4]): For each pair of coprime integers (p, q), there corresponds a 2-bridge knot, which has two kind of knot diagrams; One is a knot diagram called *Conway's normal form*, which is given by using a continued fraction expansion of p/q, and another is a knot diagram called *Schubert's normal form*, which is given directly from p and q without using a continued fraction expansion of p/q. In 1981, R. Tuler studied the first coefficient of the Conway polynomial and gave two formulas; one is a formula by means of a continued fraction expansion and another is a formula without using a continued fraction expansion. Furthermore, by using these formulas, Tuler gave an arithmetic relation about Rademacher-Dedekind homomorphism, which is an important subject in number theory. In 1985, G. Burde gave a formula for every coefficient of the Conway polynomial of 2-bridge knots by means of a continued fraction expansion of p/q. To have the formula without using a continued fraction expansion of p/q, first I studied the second coefficient of the Conway polynomial, which is an example of the Vassiliev invariant of order two called the Casson knot invariant. In [1], by using the Gauss diagram of Schubert's normal form, I gave a formula for the Casson knot invariant without using a continued fraction expansion. In [4], I extended this formula to every coefficients by introducing the map which is useful to handle the Gauss diagram of Schubert's normal form. In [3], I also gave a formula for the Vassiliev invariant of order three without using a continued fraction expansion. In [2], I studied finite type invariants of PSL(2, Z), by means of the fact that each Conway's normal form corresponds to an element of SL(2, Z). So far I have determined the invariant whose order is less than four and I gave an arithmetic relation between that invariant and the invariants of 2-bridge knots mentioned above.

Invariants of ribbon knots and their 2-fold branched covers (Publications [5, 6] and Preprints [7, 8]): In 1966, R.H. Fox and J.Milnor defined the equivalence relation on knots called *cobordance* in terms of 4-dimensional topology. The set of all knots is regarded as a group by cobordance. A knot which represents the identity element of this group is called a *slice knot*. It is a basic problem to determine the slice knots. But this problem contains many difficulties relating to 4-dimensional topology. So Fox and Milnor defined the subclass *ribbon knot*s of the slice knots in terms of 3-dimensional topology. One of the aims in investigating ribbon knots is to approach this famous problem called the slice-ribbon problem: "Are slice knots ribbon knots?" In fact, so far the knots identified as slice knots are all ribbon knots. Fox and Milnor gave the following famous property; the Alexander polynomial of slice (ribbon) knots is of the form f(t)f(1/t), where f(t) is a Laurent polynomial. To have other necessity conditions for ribbon knots I have been investigating invariants of ribbon knots. In [5], I studied the Jones polynomial of ribbon knots and I gave an explicit formula for the coefficients of the Jones polynomial of ribbon knots of 1-fusion and derived some necessity conditions for ribbon knots from this formula. In [6], to investigate the complexity of a ribbon knot, I defined the notion of the *ribbon number* as the minimal number of ribbon singularities needed for a ribbon disk bounded by the ribbon knot. This obvious measure of a knot's complexity is often hard to determine. In fact, even in a simple case of ribbon knots, its ribbon number is hard to determine. Then I estimate the ribbon number by using a formula in [5]. In particular, I determined that the ribbon number of the Kinoshita-Terasaka knot is two. In [8], Y. Tsutsumi and I investigated the ribbon number by using the genus and the crosscap number.

In 1978, Montesinos showed the 2-fold branched cover along a ribbon knot of 1-fusion is a homology 3-sphere bounding a contractible 4-manifold of Mazur type. In [7], I gave an explicit formula for the Casson invariant of a homology 3-sphere of Mazur type.

A program to find ribbon knots (Preprint [9]): I made a program to find ribbon (slice) knots in the knot table and had candidates of counter examples of the slice-ribbon problem mentioned above. Moreover, I derived some conditions to have a more precise program.