Research Plan

For the explanation of my research plan, let me here restrict myself to the studies on automorphic forms generating quaternionic discrete series. I would like to study the following two subjects in future.

1. Generalization of the Koecherer principle for any automorphic forms generating quaternionic discrete series.
2. Arithmetic of Fourier coefficients of automorphic forms on $Sp(1,1)$.

Let me explain the first theme. A remarkable feature of quaternionic discrete series, which were introduced by Benedict Gross and Nolan Wallach, is that they behave like holomorphic discrete series in spite of their non-holomorphy. Besides $Sp(1,q)$, these discrete series exist for the two classical groups, the orthogonal group of signature $(4+,q-)$ and the unitary group of signature $(2+,q-)$, and for many exceptional groups. I expect that the Koecher principle proved for the case of $Sp(1,q)$ would be able to be extended to all the other cases. I think this expectation quite natural in view of the similarity between quaternionic discrete series and holmorphic ones. I am now carrying out several observations for the case of the classical groups. An ultimately goal is to find a unified argument to study the Koecher principle for the general situation.

For the second theme let me remark that the specialists of automorphic forms often have careful look at Fourier coefficients of them when they try to find arithmetically meaningful information on the forms. In fact, the Fourier coefficients of automorphic forms had turned out to be an interesting object of the number theory since Siegel's study on quadratic forms by means of theta series on the Siegel upper half spaces (which are holomorphic Siegel modular forms). After this we have encoutered several further discoveries on the arithmetic meanings of the Fourier coefficients of automorphic forms in terms of class numbers of number fields and of intersection numbers of algebraic cycles in some arithmetic variety etc. Our interests in the Fourier coefficients are thus becoming even stronger now.

But the results obtained so far are mainly those for holomorphic forms. It is therefore quite natural to have our interest also in Fourier coefficients of non-holomorphic automorphic forms. As a first step toward it I want to find and study in detail the arithmetic meanings of Fourier coefficients of automorphic forms on $Sp(1,1)$ including the forms generating quaternionic discrete series. The result on the Fourier coefficients of the theta lift, which is explained around the end of the "The Achievement of My Researches", is obtained under the second theme (as for this it is an important problem to study the explicit relation between the Fourier coefficients and the L-functions).

To make the aforementioned research progress further, I think that we should find as many examples of the automorphic forms on $Sp(1,1)$ as possible. I have already constructed Eisenstein series and Poincaré series on $Sp(1,1)$ (more generally on $Sp(1,q)$) generating quaternionic discrete series other than the theta lift. However, it seems to me that these are not enough for my aim. Actually these forms are just standard ones. As well as these we will have to find a varieties of constructions of the automorphic forms. For instance we can consider the restriction to $Sp(1,1)$ of automorphic forms on the larger groups such as the unitary group of signature $(2+,2-)$ and the quaternion unitary group of signature $(2+,2-)$ etc. I am planning to produce plenty of examples of the Fourier coefficients for the forms obtained in the various manners in order to find arithmetic meanings of the coefficients.

One reason why the researches on the arithmetic of automorphic forms on Hermitian symmetric domains are successful, is that a locally symmetirc domain, which is defined as a quotient of a Hermitian symmetric domain by an arithmetic group, has a geometric model called a "canonical model" over an arithmetic ring such as a number field or its ring of integers etc. But we have not yet found how to construct such model (or something like it) for the case of $Sp(1,1)$, or more generally for semisimple groups or Riemannian symmetric spaces of non-Hermitian type. I hope that the significance of our research mentioned above can be explained in terms of the extension of the arithmetic researches on automorphic forms to non-holomorphic ones on the groups or the symmetric spaces of non-Hermitian type.