The Achievements of My Researches

My researching field is the number theory. I mainly study the theory of automorphic forms. As the most classical example of an automorphic form we have an elliptic modular form. It is defined as a holomorphic function on the complex upper half plane, equipped with lots of "symmetry", i.e., the automorphy with respect to an arithmetic subgroup of the special linear group $SL_2(\mathbb{R})$ of degree two. So far its various generalizations have been considered and studied. We can view automorphic forms as functions on simisimple Lie groups (for instance, elliptic modular forms are also regarded as functions on $SL_2(\mathbb{R})$), whence they are often investigated in terms of the representation theory of the semisimple groups. Besides the theory of automorphic forms, the number theory consists of many branches: Arithmetic Geomerty, Algebraic Number Theory and Analytic Number Theory etc. These interact with each other to lead to the current highly developped status of the number theory. In fact, such interactions have brought us great achievements. As the most typical examples we have the solution of Fermat's Last Theorem (settled around the end of the last century) and the recent great progress toward the Sato-Tate conjecture etc. In the various great achievements of the number theory such as these, automorphic forms have often played crucial roles. Of course, the theory of automorphic forms have been studied in its own interests, and it have been also providing the number theory with powerful tools of investigations.

In a wide view, the aim of my research is to contribute to the further advancement of the studies on holomorphic automorphic forms, which have been mainly investigated by the specialists until now, and to extend the scope of the researches on automorphic forms to non-holomorphic ones. Under these motivations I have been studying the following subjects:

1. Fourier expansion of holomorphic automorphic forms on a classical tube domain (a generalization of the Siegel upper half space).
2. Researches on automorphic forms on the quaternion unitary group $Sp(1,q)$ of signature $(1+, q-)$ in terms of the Fourier expansion, explicit constructions of automorphic forms, arithmetic of L-functions and Fourier coefficients etc.

Now I am studying the second one with my greatest interest. The automorphic forms in (2) can be viewed as real analytic forms on a quaternion hyperbolic space, which is the Riemannian symmetric space corresponding to $Sp(1,q)$. An interesting aspect of the automorphic forms is that they behave like holomorphic forms in spite of their non-holomorphy. Actually I have succeeded in proving the "Koecher principle" for the automorphic forms on $Sp(1,q)$ above, i.e., they are automatically of moderate growth. It has been known that holomorphic automorphic forms except elliptic modular forms satisfy this property. The Koecher principle can be regarded as the "Haltogs's continuation theorem" for automorphic forms. Namely we can say that the automorphic forms on $Sp(1,q)$ generating quaternionic discrete series provide examples of real analytic functions on the quaternion hyperbolic space (which is not complex analytic) satisfying the property similar to the Hartogs theorem. In addition, such similarity to holomorphic forms suggests us that the forms on $Sp(1,q)$ would be relatively comfortable to study among non-holomorphic automorphic forms. In general, the investigations of non-holomorphic forms are very difficult since we can not use the complex analysis etc. But I hope that the automorphic forms mentioned just above give us a first step toward extending the studies of automorphic forms to non-holomorphic forms. Furthermore let us note that the quaternion hyperbolic spaces are also the objects of reseaches for the geometry, more precisely the hyperbolic geometry. Every detailed studies related to the spaces will be targets for that geometry. I think it interesting to try to find the significance of our researches on the automorphic forms in terms of this background.

Finally let me inform you that we have the further recent progress on the arithmetic of the automorphic forms on $Sp(1,1)$ by a joint work with Atsushi Murase, a professor of Kyoto Sangyo University. To be more precise, we have obtained an explicit formula for the Fourier expansion of the automorphic forms constructed by a theta lift from pairs of elliptic cusp forms and automorphic forms on a multiplicative group of a division quaternion algebra. We have succeeded in writing the Fourier coefficients in terms of the toral integrals of the two forms in the pre-image of the lift. Such integrals are expected to relate to L-functions, thus we can say that the appearance of such integrals in the formula reflects the arithmetic significance of the coefficients.