Research plan

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(1) I want to construct the other CAP forms.
I constructed examples of CAP forms for a quaternionic inner form G of $Sp(2)$, but there are other CAP forms of G. For example, in the case of inner form version of Saito-Kurokawa representations I could construct them if an infinite dimensional irreducible cuspidal representation π of $GL(2,\mathbb{A})$ satisfies that its standard L-function $L(s,\pi)$ does not vanish at $s = 1/2$. However I could not construct them if $L(1/2,\pi) = 0$. In this case they will be constructed by the theta lifts from the two-fold cover of $SL(2)$ considered by Piatetski-Shapiro. Also there should exist an inner form version of CAP forms with respect to the non-Siegel parabolic subgroup which are given by Soudry. They will be constructed by the theta lifts from the unitary group of a rank 1 skew-hermitian space. From the method of the construction and the failure of the Hasse principle they should not satisfy the multiplicity one property. In case of $GSp(2)$ Piatetski-Shapiro and Soudry characterize CAP forms using L-functions. I will consider the characterization of CAP forms of G using L-functions similarly. For this I have to consider the definition of L-functions for G.

(2) I want to describe the non-vanishing of Yoshida lifts and Arakawa lifts.
The inner form version of the Saito-Kurokawa representations were constructed by the theta lifts of irreducible cuspidal representations of the special unitary group $SU(V)$ of rank 2 skew-hermitian space V. For V there is a quaternion algebra B so that $SU(V)$is realized by

$$\{(b,\widetilde{b}) \in B^\times \times \widetilde{B}^\times \mid \nu_B(b) = \nu_{\widetilde{B}}(\widetilde{b})^{-1}\}/\{(z,z^{-1}) \mid z \in k^\times\}.$$

Here $\widetilde{B}$ is a quaternion algebra which coincides with $B \cdot R$ in the Brauer group of k, and $\nu_B, \nu_{\widetilde{B}}$ are the reduced norms of $B, \widetilde{B}$, respectively. By this an irreducible cuspidal representation of $SU(V_\mathbb{A})$ may be regarded as the tensor product of irreducible cuspidal representations of $B_\mathbb{A}^\times$ and $\widetilde{B}_\mathbb{A}^\times$. In case of CAP forms of G, it is suffices to consider irreducible cuspidal representations of $SU(V_\mathbb{A})$ of a form $\pi \otimes \mathbf{1}$. We will consider more general irreducible cuspidal representations of $SU(V_\mathbb{A})$ of a form $\pi_1 \otimes \pi_2$ where π_2 is not necessary to be $\mathbf{1}$. The theta lift from this representation is called a Yoshida lift or an Arakawa lift. This lift is well-defined as theta integral, but its image may vanish. The necessary and sufficient condition of the non-vanishing has not been known yet. I want to describe this condition.