Plan of the work

I want to study the relation of the derivative of the p-adic L-functions and Euler systems. First, I will begin with the study of the behavior of zero points of the Kubota-Leopoldt p-adic L-functions. Ferrero-Greenberg showed that the Kubota-Leopoldt p-adic L-function $L_p(s, \chi)$ have at most a simple zero at $s = 0$. To prove this, they gave a certain formula for the derivative of $L_p(s, \chi)$ at $s = 0$ by using Morita's p-adic Γ-function. On the other hand, Gross-Koblitz gave an explicit formula of Gauss sums in locla fields in terms of p-adic Γ-function. I am interested in the local property of Gauss sums because it is related to the plus part of the ideal class groups. The result of Gross-Koblitz asserts that we can get the local property of Gauss sums from p-adic Γ-function. I want to study this p-adic Γ-function in detail, and make this result more precise. I think that this is very important because the behavior of zero points of p-adic L-functins is not well known. As an application of this study, I want to get a new reflection theorem of ideal class groups. The reflection theorem says that the difference of the plus part and the minus part is "small". I want to determine the difference of both sides. Further by study of local behavior of Gauss sums, I also study ideal class groups of towers of cyclotomic number fields and approach Greenberg conjecture.

I am also interested in the relation of these studies and Knot theory. As Alexander polynomials and Jones polynimials are defined to be invariants of Knots, Iwasawa polynomials are defined to be invariants of ideal class groups. These polynomials relate to important conjectures (for examples, Vandiver conjecture, Greenberg conjecture), and hence these polynomials are very important. As we can calculate the Jones polynimial of a Knot, we can also calculate the Iwasawa polynomials for each number fields. The higher polynomials which come from Euler systems give more imformation to ideal class groups. I want to give a definition of higher Alexander polynomials and Johnes polynomials, and define new invariants.