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Plan of research

The following is the present background of my studies.

[Background 1]

In my thesis [2], I introduced the P-tableau as a conserved quantity of BBS. On the other hand, some other types of conserved quantities are reported by a work of Torii and Nagai and by the thesis [1]; they are related with the soliton contents (the number of solitons per length). Since it is impossible to read off any data related with soliton contents from the P-tableau, I conjecture that Q-tableaux still contain some conserved elements. It is important to persue detailed studies on conserved quantities related with soliton contents. In particular, I want to extract such data from the P-tableau and Q-tableaux introduced in [2]. Although the shape of P and Q is clearly conserved, it is also important to find the combinatorial meaning of this fact.

[Background 2]

There is no researcher studying the tropical approach (below (iii)) to the time evolution of the Q-tableau. It is one of the important subjects in the investigation of the tropical approach itself.

[Background 3]

As for the generalization of BBS, Kuniba and others carried it out in terms of crystal theory. But there is no complete understanding yet from the viewpoint of tableaux.

The purpose of my research is to study BBS, soliton cellular automata (especially the time evolution of the Q-tableau) in terms of pure combinatorics, and to solve problems about conserved quantities, corresponding integrable systems and their generalization. I also want to be able to use computer perfectly for my research. I now explain my plans in some detail as follows:

(i) Extraction of conserved quantities from the Q-tableau: As mentioned above, there is a sign of existence of some conserved quantities in Q-tableaux. Y. Yamada has a stock of experimental results on this problem together with his joint workers (Kuniba, Okado, and Takagi); I want to work jointly with them and solve this problem. I will extract some new conserved quantities by using various functions adopted for tableaux combinatorics, and dividing Q-tableau into the conserved part and the evoluted part.

(ii) Relation of various conserved quantities : I will study the relation between the conserved quantity of (i) and those obtained earlier by Torii, Nagai and others.

(iii) Tropicalization of the time evolution Q-tableau: As for above (i), the time evolution of the Q-tableau can be described combinatorially by a max-plus algebra. I will pass from these subtraction-free rational functions to piecewise linear transformations (tropical approach), and construct the corresponding integrable system. I will also apply this idea to the generalized BBS, and construct its hierarchy.

(iv) Generalization of the model (extension to root systems) : As for the BBS associated with an affine Lie algebra, at least of classical type (B, C, D), I will describe BBS by means of tableaux, and generalize the results of type A to other root systems. I will also try applying my idea to the periodic BBS with finite degrees of freedom.

(v) Generalization of the model (by the shifted tableaux) : From a slightly different point of view, I want to introduce some BBS which has a shifted tableau as its conserved quantity. As for the shifted tableau, there is a correspondence similar to RSK; therefore, it is an interesting theme to extend the result of previous research (Research results 4) to this (shifted) case.