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Research results

In 1980s, Wolfram and others proposed the most fundamental cellular automaton, and indicated that cellular automata have some solitonical solutions. In Japan, in 1990, Takahashi and Satsuma proposed a cellular automaton called the "soliton cellular automaton". It has a remarkable property that all patterns behave like as solitons under the boundary condition to be zero at infinity. They represented and extended this soliton cellular automaton as a dynamical system of balls moving in an array of boxes. This system is called the "box-ball system (BBS)". Kuniba, Takagi, Tokihiro, and others have studied physical aspects of the BBS in relation with soliton equations and solvable lattice models. Recently, the BBS is studied in terms of crystal bases and representation theory, and it has been payed attetion to in various fields of mathematics as well. I have been studying the BBS intensively in terms of combinatorial representations by tableaux. The following is the list of my achievements.

(1) Proof of the equivalence between two different time evolutions of BBS (Master's thesis)
In 1999, while the relation between crystal bases and BBS proposed by Takahashi-Satsuma was developed, it was an interesting problem to establish the equivalence of two different algorithms of time evolution. I gave a combinatorial proof for this equivalence without theories of ultra-discrete soliton equations and crystal bases.

(2) Construction a conserved quantity of BBS by the Knuth equivalence (Master's thesis)
In 1999, from the equivalence above, I recognized that the time evolution of BBS conserves the Knuth equivalence. I also constructed the P-tableau by bumping from every state of BBS, and proved that it is conserved under the time evolution of BBS.

(3) Relation between BBS and crystal bases (Thesis [1])
Around the same time as above, the relation between BBS and crystal bases was discovered simultaneously by some Japanese groups (Fukuda-Okado-Yamada, Hikami et al., Hatayama et al.). In a joint work with Okado and Yamada, I showed in particular the relation between the phase shift of scattering and the energy function.

(4) Algorithm of time evolution for the Q-tableau in BBS (Thesis [2])
After that, I developed the idea mentioned in (2). When a state of BBS is transformed into a pair of tableaux (P,Q) through the Robinson-Schensted (RS) correspondence, the P-tableau gives rise to a conserved quantity of the BBS. Also, the algorithm of time evolution can be described combinatorially only in terms of the Q-tableau. The rule of evolution for the Q-tableau provides an effective algorithm for BBS. It is an important subject in my research to analyze the algorithm of time evolution for the Q-tableau.

(5) Generalization of BBS by the RSK correspondence (Thesis [2])
I generalized the way to attach a pair of tableaux (P,Q) to each state, by using the Robinson-Schensted-Knuth (RSK) correspondence in place of the RS correspondence. By doing so, I applied the idea mentioned above naturally to the generalized BBS with more degrees of freedom, obtained by increasing the number of "colors of balls" and the "capacity of each box". However, it is an open problem to generalize this approach from the RSK correspondence to the BBS associated with affine Lie algebras other than those of type A.