My recent work

Naoko Kamada

In 1988, E. Witten proposed a topological invariant of a 3-manifold, the Quantum G-invariants, by using a compact simple Lie group G. N. Yu. Reshetikhin, V. G. Turaev, R. Kirby and, P. Melvin gave a mathematical proof for existence of the invariant and found a relationship between the Quantum $PSU(2)$ invariant of a 3-manifold obtained from S^3 by surgery along a link and its Jones polynomial. H. Murakami showed a relationship between the Quantum $PSU(2)$ invariant and the Casson-Walker invariant of a rational homology 3-sphere. In [1], I gave the same relationship between the Quantum $PSU(2)$ invariant and the Casson-Walker invariant of a 3-manifold whose $Z/5Z$ Betti number is positive.

In 1985, V. F. R. Jones defined the Jones polynomial, via study of certain finite dimensional von Neumann algebras, which is one of the most important discovery in knot theory. On the other hand, L. H. Kauffman defined a regular isotopy invariant of a link diagram on S^2, called the Kauffman bracket polynomial, by using a state model. He proved that his invariant, after normalization with the writhe effect, is equal to the Jones polynomial. In [2], I generalized the Kauffman invariant for a link diagram on S^2 into that for a link diagram on any closed oriented connected surface F, and generalized the Jones invariant for a link in S^3 (R^3 or $R^2 \times R$) into that for a link in the product space $F \times R$ of F and the real line R. The Kauffman/Murasugi Theorem—Any proper alternating connected link diagrams of a link in S^3 have the same number of double points—was generalized for a link in the thickened surface $F \times R$.

In the late 1990's L. H. Kauffman introduced virtual knot theory. It is a generalization of knot theory, which was motivated by the study of knot diagram in a closed oriented surface and abstract Gauss codes. A virtual knot diagram is obtained from a knot diagram by replacing some crossings with virtual crossings. He generalized the Jones polynomials of knots to those of virtual knots. M. Goussarov, M. Polyak, and O. Viro studied the Vassiliev invariant by using virtual knots. D. S. Silver and S. G. Williams characterized the knot groups of virtual knots. J. Sawollek applied the invariant of a link in thickened surfaces due to F. Jaeger, L. H. Kauffman and H. Saleur to virtual knots (called the JKSS-invariant.) An abstract link diagram is a pair of a compact oriented surface and a link diagram on it such that the diagram is a deformation retract of the surface. In [3], S. Kamada and I showed that there is a natural bijection between virtual knots (links) and abstract links. Using abstract links, we gave a geometric interpretation of the fundamental groups of virtual knots. Jones polynomials of virtual knots are quite different from those of classical knots. In [4], it is proved that Jones polynomials of virtual knots that are presented by checkerboard colorable diagrams have a certain property that classical knots have. A skein relation among the Jones polynomials of three virtual knot diagrams was given in [5]. In [6], a skein relation of the Jones polynomials of another kind of three virtual knot diagrams was given, which is complementary to the previous one.