**Abstract of the work**

The Iwasawa polynomial in Iwasawa theory is very important, because it constructs the $p$-adic $L$-function, and it is related to ideal class groups by Iwasawa main conjecture. This polynomial is similar to Alexander polynomial in Knot theory, hence the connection between Iwasawa theory and Knot theory has been studied recently. My main work is the application of Euler systems for the $p$-adic representation $T = \mathbb{Z}_p(1)$ and the family of abelian extensions over $\mathbb{Q}$, that is, it is the application of Euler systems of cyclotomic elements, Gauss sums and Stckelberger elements to Iwasawa theory. It is well known that we can know the bound on the order of Selmer groups by using Euler systems , but other application of Euler systems is not so known. We can think Euler systmes to be higher Iwasawa polynomials, because Iwasawa polynomials is constructed by Stickelberger elements or cyclotomic elements. The summaries of my papers are as follows.

**In the paper: The Iwasawa Main conjecture and Gauss sums**, I gave a new proof of the Iwasawa main conjecture by using the Euler system of Gauss sums. This conjecture gives a relation between ideal class groups of towers of cyclotomic number fields and certain $p$-adic $L$-functions. First proof for this conjecture was given by Mazur and Wiles by using modular forms, and after Kolvagin introduced Euler system, Rubin gave a completely different proof by using the Euler system of cyclotomic units. In this paper, I used the Euler system of Gauss sums and directly study the minus part of the ideal class groups. **In the paper: Local units and Gauss sums**, I determined the structure of a certain module which is related to the plus part of the ideal class groups in terms of the divisibility of Gauss sums in local fields. We can get information on the minus part of the ideal class groups by Gauss sums in global fields, on the other hand, Gauss sums in local fields is related to the plus part. Iwasawa gave a necessary and sufficient condition of Vandiver's conjecture using the local property of Gauss sums. The result in this paper is a generalization of Iwasawa's rerult (,and Ichimura-Hachimori's result). **In the paper: Notes on the structure of the ideal class groups of abelian number fields**, I gave explicit formulae of higher annihilators which appear in the Euler systems of cyclotomic units and Gauss sums, and using these formulae, I calculated some examples of ideal class groups. It is well known that Stickelberger elements are good annihilators of the minus part of ideal class groups. For the plus part, Thaine constructed a good annihilators by using cyclotomic elements. Higher annihilators enable us to determine the structure of ideal class groups. In this paper, I reformulated the family of higher annihilators to compute the structure of ideal class groups numerically. Furthermore recently, I improved on this result and got better algorithm for caluculation of ideal class groups (with T.Fukuda).