# Research Plan

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I have studied invariants and local moves on knots and links. I hope that I promote the study.

Recently V. A. Vassiliev introduced a new knot invariant called a Vassiliev invariant. A Vassiliev invariant of order $n$ for knots can be extended to that for links. The invariants include a lot of invariants which have been studied.

On the other hand, K. Habiro defined a new local move called a $C_n$-move. A $C_n$-move is closely related to Vassiliev invariants. It is shown that if two links are related to by a finite sequence of $C_{n+1}$-moves, then they are $V_n$-equivalent, where we say that two links are $V_n$-equivalent if they are not distinguished by any Vassiliev invariant of order less than or equal to $n$.

In the case of knots, M. N. Gusarov and Habiro independently showed that two knots are related by a finite sequence of $C_{n+1}$-moves if and only if they are $V_n$-equivalent.

The above result shows that a $C_{n+1}$-move completely corresponds to $V_n$-equivalence. But for links, the situation is different. There exist two links which are $V_n$-equivalent but not transformed into each other by a finite sequence of $C_{n+1}$-moves for $n \geq 2$. If two links are transformed into each other by a finite sequence of $C_n$-moves, then they are $V_n$-equivalent. So we would like to know which invariants are needed except for Vassiliev invariants of order less than or equal to $n$ to describe a neccesary and sufficient condition to be related by $C_{n+1}$-moves. For this problem, we have some partial answers for special links. In general case, the problem is open. It seems to be difficult and complicated, but I think I should try.

Considering to generalize Gusarov-Habiro's theorem, we have another approach. That is to show the geometrical condition for two links to be $V_n$-equivalent. For this problem, I obtain the results for $n = 2, 3$. I will continue the research.

I am also interested in inducing an algebraic structure in the set of links. Because considering the quivalence relation of knots generated by $C_n$-moves, the equivalence classes of knots forms a group for each $n$ where a group operation is induced by the operations of taking connected sum. This is one of the keys to prove that $V_n$-equivalence implies that two knots are deformable by $C_{n+1}$-moves. Generally the connected sum for links is not well-defined up to ambient isotopy. Hence I am thinking of defining a reasonable connected sum for links.