Summary of Research

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A study of local moves is one of subjects in the knot theory. A local move means a replacement of a part of a knot or link diagram.

I have been interested in local moves and studied it. In [1], I defined a new local move called an n-gon move for an integer n greater than 2, and showed that an n-gon move is a kind of unknotting operations for each n. Furthermore I proved that there exists an integer n for any knot K such that K can be transformed into a trivial knot by a single n-gon move.

Recently V. A. Vassiliev introduced a new knot invariant called a Vassiliev invariant of order n. He regards a knot as a smooth map from R^1 into R^3 and consider the cohomology of the space of the smooth maps. Then he obtains an invariant corresponding to a positive integer n. It is known that we can define a Vassiliev invariant of order n for links as for knots.

Vassiliev invariants include a lot of invariants which have been known. For example, the coefficient of z^n of the Conway polynomial of an oriented link L (denoted by $a_n(L)$) and the n-th derivative at $t = 1$ of the Jones polynomial $V_L(t)$ of L (denoted by $V^{(n)}(L)$) are Vassiliev invariants of order n. All of the quantum invariants (including homfly polynomial and Kauffman polynomial) are Vassiliev invariants.

K. Habiro defined a local move called a C_n-move for a positve integer n. For each n, C_n-move is defined as an element of a finite set of local moves, but the set is generated by a C_n-move. So we consider the C_n-move. A C_n-move is related to Vassiliev invariants. M. N. Gusarov, T. Stanford and Y. Ohyama showed independently that if two links L and L' are transformed into each other by a finite sequence of C_{n+1}-moves, $v(L) = v(L')$ holds for any Vassiliev invariant of order less than or equal to n.

From this result, we can say that if two links L and L' are transfomed into each other by a finite sequence of C_n-moves, $a_k(L) = a_k(L')$ and $V^{(k)}(L) = V^{(k)}(L')$ hold for $k = 1, 2, \ldots, n-1$. In [2], I studied that the relation between $a_n(L)$ and $a_n(L')$, and $V^{(n)}(L)$ and $V^{(n)}(L')$ for two links L and L' which are transformed into each other by a C_n-move. Using this result, we can prove easily that two links cannnot be transformed into each other by a finite sequence of C_n-moves. Considering knots, a C_{n+1}-move completely corresponds to Vassiliev invariants of order n. Gusarov and Habiro showed that two knots K and K' are transformed into each other by a finite sequence of C_{n+1}-moves if and only if $v(L) = v(L')$ holds for any Vassiliev invariant of order less or equal to n.

In the case of links, the above result does not hold for $n \geq 2$. In [3] I define an SC_n-move as a special C_n-move and showed that for $n = 2, 3$, two links L and L' are transformed into each other by a finite sequence of C_{n+1}-moves and SC_n-moves if and only if $v(L) = v(L')$ holds for any Vassiliev invariant of order less or equal to n (where if $n = 3$, restrict to 2-component links).

Furthermore I study the relation between polynomial invariants of two links which are transformed into each other by an SC_n-move in a paper in preparation.

Papers

[1] *Unknotting operations of polygonal type*, Tokyo Jour. Math.,Vol. **15** (1992), 111-121.
[2] *C_n-moves and polynomial invariants for links*, Kobe Jour. Math.,Vol. **17**, (2000), 99-117.
[3] *C_n-moves and V_n-equivalence for links*, preprint (2003).