RESEARCH PROJECTS

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I would like to continue to study the transformation group theory. Especially I will put stress on the GKM-graph which has been studied in the paper (4) in List of my papers (and Part 2 of my Ph.D thesis). This area has been researched on actively in recent years and one can expect to built a new bridge between topology and combinatorics.

As a new class of a GKM-graph I introduced the notation of a hypertorus graph in the paper (4), as I have already written in Summary of my research. Then I got the result about the equivariant graph cohomology ring $H_T^*(\Gamma)$ under some assumptions on the hypertorus graph (which contains the GKM-graph defined by the hypertoric variety), but I hope this result can be generalized (to the case which contains the GKM-graph defined by the cotangent bundle of the torus manifold). As an application of its generalization, I also hope we can get a combinatorial description of $H_T^*(\Gamma)$, where Γ is not only a torus graph in the paper of Maeda-Masuda-Panov but also a generalized torus graph which is possible to have a 'leg' (a half line going out from one vertex). From these points I expect that the class of the hypertorus graph essentially contains all classes of the GKM-graph whose equivariant cohomology can be described by some combinatorial date.

Moreover I defined a quaternionic torus graph which is generalized class of the hypertorus graph in the paper (4). This graph contains a GKM-graph, which is not only defined by a hypertoric variety or the cotangent bundle of a torus manifold but also defined by the quaternionic projective space $\mathbb{H}P(n)$ or the comlex quadric Q_{2n}. This property has a possibility that we can define a very interesting class of manifolds, that is I hope we can define a class of manifolds that we can call a *quaternionic toric manifold*. The motivation of this research is the research about the quasi-toric manifold which was defined by Davis and Januszkiewics in 1991. The quasi-toric manifold is a topological construction of the toric manifold which is an important class in the algebraic geometry. There is a complex projective space $\mathbb{C}P(n)$ with the T^n-action as a typical example of the (quasi-)toric manifold. Since the circle S^1 is a mximal compact subgroup of $\mathbb{C}^*$, we can consider the (quasi-)toric manifold is a complex. Davis and Januszkiewics also defined a small cover, which is a real part of toric manifold in one sense, as the important class in the same paper. There is a real projective space $\mathbb{R}P(n)$ with ${\mathbb{Z}_2}^n$-action as a typical example of it. Since we can assume $\mathbb{Z}_2$ is a maximal compact subgroup of $\mathbb{R}^*$, we can think the small cover is a real. So it is natural to consider that we might be able to construct the class which corresponds to the quaternion of the toric manifold (quaternionic toric manifold). If we define a quaternionic toric manifold and its group action, then we naturally hope that it contains a quaternionic projective space $\mathbb{H}P(n)$ with $Sp(1)^n$-action as a typical example. $Sp(1)$ is a maximal compact subgroup of $\mathbb{H}^*$. In 1995 R. Scott tried to define such class by the similar construction of Davis and Januszkiewics under this motivation. However the object coming from his research did not have an $Sp(1)^n$-action. I think this result is difficult to say that he succeeded to construct a class which corresponds to the quaternion of the toric manifold, from the transformation theoretical view point. Therefore the second project of my research is to define such a class and study how different our class is from toric manifolds and small covers. If we construct a manifold and T^n-action from our quaternionic torus graph, and if its action can be generalizad to $Sp(1)^n$-action, then we can hope that the quaternionic toric manifold appears even if we consider from the transformation theoretical view point.

Besides I will discover relationships between topology and combinatorics by studying a GKM-graph deeply. In the future I will also discover topological invariants of non-compact transformation groups by using wisdoms and techniques from the above researches.