Achievement of the Research, Hiro-aki Narita

My researching field is the number theory. I am mainly interested in the arithmetic theory of automorphic forms. Until now I have given several theories of Fourier expansions of automorphic forms and their applications. In the theory of automorphic forms Fourier expansions provides a fundamental and useful tool of the investigation. Actually it holds various information on analytic or arithmetic properties of automorphic forms, for instance, it is very useful to study automorphic L-functions, which is one of central concerns in the arithmetic theory of automorphic forms. The Fourier expansions I have investigated are the following two: (1) the Fourier expansion of holomorphic automorphic forms on tube domains with respect to the minimal parabolic subgroup, (2) the Fourier expansion of automorphic forms on the quaternion unitary group $Sp(1,q)$ generating quaternionic discrete series. Recently I have obtained further results for the latter automorphic forms based on the Fourier expansion. The automorphic forms on $Sp(1,q)$ generating quaternionic discrete series are one of my main concerns now. In the following I shall explain my results obtained for the research of these automorphic forms.

As the first application of the Fourier expansion, I have proved the "Koecher principle" for the automorphic forms when $q > 1$. Namely I have shown that they automatically satisfy the moderate growth condition (this does not hold for the case of $q = 1$). This is an interesting aspect of the forms. In fact, this property is known for holomorphic automorphic forms of higher degree but we should note that our automorphic forms on $Sp(1,q)$ are non-holomorphic. In other words, we can say that they behave like holomorphic automorphic forms in spite of their non-holomorphy. As far as I know there are quite a few examples of non-holomorphic automorphic forms satisfying the principle (Don Zagier pointed out that Hilbert Maass forms also satisfy the Koecher principle in the above sense).

As the second application, I have provided explicit constructions of our automorphic forms. More precisely I have given two constructions as follows: (1) Eisenstein series and Poincaré series (2) a "theta lifting" from elliptic cusp forms. We have the Eisenstein series (resp. Poincaré series) by considering some infinite series constructed from the function appearing in the constant term (resp. a cuspidal term) of the Fourier expansion. Our theory of the Fourier expansion implies that these two kinds of series provide a basis for the space of the forms on $Sp(1,q)$ generating a quaternionic discrete series. On the other hand, the theta lifting construction was given by Tsuneo Arakawa, who is the initiator of the research of our automorphic forms. When $q = 1$ he proved that the images of the lifting are the automorphic forms generating quaternionic discrete series. However, Arakawa did not deal with the case of $q > 1$. In order to prove this remaining case, I computed the Fourier expansion of the forms lifted from elliptic Poincaré series. Thereby I have resolved the case of $q > 1$.

Very recently Atsushi Murase (from Kyoto Sangyo University) and I have jointly studied arithmetic properties of the theta lifting. More precisely, for the case of $q = 1$, we have reformulated Arakawa's theta lifting as a lifting from a pair of elliptic cusp forms and automorphic forms on a definite quaternion algebra to automorphic forms on $Sp(1,1)$ in the adelic setting, and studied the action of Hecke operators on the lifted forms. Then we have proved that such a modified lifitng satisfies a good commutation relation of Hecke operators acting on elliptic cusp forms, automorphic forms of the quaternion algebra and our automorphic forms. By virtue of this result we have determined all the non-Archimedean local factors of the automorphic L-function (more precisely, the spinor L-function) for the lifting using the Hecke eigenvalues of elliptic cusp forms and automorphic forms on the quaternion algebra. We have found several interesting properties of this L-function. For instance, its local factors at unramified Archimedean places decomposes into a product of local factors of L-functions for elliptic cusp forms and automorphic forms on a definite quaternion algebra. In some case, we have seen that all the local factors for ramified Archimedean places coinside with local factors of L-functions for elliptic cusp forms.