Plan of research

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In knot theory and spatial graph theory, it is one of aims to make the classification table. Depending on the kinds of equivalence relations various classifications are obtained. A classification by local moves is one of them. From the viewpoint of starting from a rough classification and gradually giving more precise classification, we would like to give classifications by some local moves by using invariants which are easily calculated from regular diagrams. In [2], we extended the concept of boundary links to spatial graphs. We defined a spatial embedding of a graph with spanning surfaces corresponding to the set of all knots in the embedding as a boundary spatial embedding. A collection of spanning surfaces is a set of connected, compact and orientable surfaces with disjoint interiors bounded by knots in the embedding. We gave a classification of boundary spatial embeddings up to self-pass equivalence by Arf invariants. However, this definition limits graphs which have boundary spatial embeddings to planar graphs satisfying certain conditions. Hence we attempted the generalization of the definition and try to extend the classification which we obtained to whole graphs. We got an idea from the concept of a locally unknotted embedding introduced by K. Kobayashi and we worked on defining standard embedding of graphs in [3]. We gave the upper bound of the number of spanning surfaces and constructed a spatial embedding which realizes the upper bound by disks. Hence we expect that our result can be enhanced naturally by adopting the spatial embedding which realizes the least upper bound with surfaces as a new definition of boundary spatial embeddings. Though spatial embeddings which realize the least upper bound by disks do not have uniqueness which is as strong as the unknot and the unlink have, we think they can be thought as one of candidates of a standard embedding. We will consider whether it is appropriate to call these embeddings standard embeddings.

We used invariants which do not change by local moves to classify spatial graphs in [1] and [2]. On the other hand, there are approaches using the degree of differences of a certain invariant which some local move causes to examine topological structures. We want to apply them to the research of global topological structures by paying attention to some invariants which can be changed by some local moves and by considering how the local moves changes the invriants and what information the differences have. From this view point, we try to classify braids by exchange moves. By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link L which is a closed n-braid ($n = 1, 2$ or 3) has only a finite number of conjugacy classes of n-braid representatives in the n-braid group. Moreover Birman-Menasco proved that if an oriented link has infinitely many conjugacy classes of n-braid representatives in B_n, the finitely many conjugacy classes divide into finitely many equivalence classes by exchange moves. For a given link, it seems that it is difficult to concretely decide the classes of braids which represents the link and which are transformed into each other by exchange moves. However, it is a very interesting problem because it can be used to judge whether the braid represents given knot. In preprint [6], for any knot (resp. any link satisfying certain conditions) represented as a closed n-braid ($n \geq 3$) we construct an infinite sequence of pairwise non-conjugate $(n+1)$-braids representing the knot (or the link). We used Conway polynomials to show that the braids in our sequence are pairwise non-conjugate. Any adjacent braids in our sequence are related by an exchange move which corresponds to a certain local move on links obtained from these braids. This local move alters the third coefficients of Conway polynomials by $2 - n$. From this result, it seems that the approach using rewriting from braid theory to knot theory is possible to give a classification by exchange moves. For instance, we start from the 3-braid group that has already been classified, to examine how these conjugacy classes disperse in each braid group as the class by exchange moves and to describe how many classes including infinite sequences there are. Because it seems to be effective to examine behavior of exchange moves in our sequences to determine the classes generated by exchange moves. We try to obtain some information about braids in our sequence from the difference of coefficients caused for a local move of the links obtained from these braids. At the same time, we will try to obtain a further improvement of our result.