Plan of Research

Ryosuke Yamamoto

Giroux's one-to-one correspondence between open book decompositions and contact structures of closed oriented 3-manifolds is playing an important role to study both structures. On the study of open book decompositions, the solution of Harer's conjecture is its great contribution. On the study of the contact structures, a characterization of contact structures through corresponding open book decompositions has become a very powerful tool.

I plan the following researches:

Overtwisted contact structures and open books

Recent research on contact structure of 3-manifold is based on dividing them into two classes, tight contact structures and overtwisted ones. I showed the following as mentioned in "Summary of my research": "A given open book decomposition is corresponding to an overtwisted contact structure if and only if we may positively stabilize it to be an open book decomposition with Stallings twist." In the proof of this, I showed a technique of positive stabilizaitions detecting overtwistedness. I expect that analysing this technique leads to a criterion of overtwistedness without stabilizations.

Monodromy maps of open books corresponding tight contact structures

Monodromy map of an open book decomposition for a closed oriented 3-manifold is represented by a product of Dehn twists on the fiber surface. It is known that an open book decomposition is corresponding to a tight contact structure if its monodromy can be represented a product of only positive Dehn twists, and also known that the reverse is not true. I will investigate negative Dehn twists on a fiber surface to understand when they break the tightness of corresponding contact structures.

Murasugi decomposition of fiber surface and contact structure

It has been a long-pending problem to find a criterion for whether a fiber surface is decomposable with respect to Murasugi sum. I plan to study for characterization of the contact structure on S^3 supported by an open book decomposition with fiber surface which is not decomposable with respect to Murasugi sum.

Knots in S^3 with lens surgery

I will try to solve the following conjecture:

"A knot in S^3 admits a Dehn surgery yielding a lens space if and only if the knot is in Berge's list."

The Berge's list is a list of "double-primitive" knots introduced by Berge. It is known that every knot in Berge's list is fibered and has tunnel number one. In other words, the knot gives S^3 an open book decomposition and a Heegaard splitting of genus 2. To understand how the fibration structure of the knot exterior works to yield a lens space by the surgery, I plan to analyze a foliation on the genus 2 Heegaard surface which is formed by the intersection of the surface with plains of the contact structure supported by the open book decomposition. I expect that it would develop a new approach to study knots with lens surgery.