Summary of my research

Ryosuke Yamamoto

Every oriented closed 3-manifold has a fibered knot/link and one can decompose the manifold into a tubular neighbourhood of the knot/link and a surface bundle over S^1. This decomposition is called an open book decomposition of the manifold.

Many results have been obtained from a viewpoint that structures of a fiber surface describe the structure of the open book decomposition. In particular, it is known due to Gabai that a decomposition of a fiber surface with respect to Murasugi sum suits the fibration on the exterior of the fibered link. For example, a surface R obtained by Murasugi sum of two surfaces R_1 and R_2 is a fiber surface if and only if both R_1 and R_2 are. I studied the structure of fiber surfaces in S^3 with respect to Murasugi sum and obtained the following results.

"Hopf band" is a fiber surface in S^3 with first Betti number 1, which is the simplest fiber surface except 2-disk in S^3. We may say that a "Hopf plumbing" i.e., a fiber surface which can be decomposed into some Hopf bands with respect to Murasugi sum, has a basic strucuture of fiber surface. It is known that a fiber surface of a fibered alternating link is a Hopf plumbing. Then the following natural question comes up: What kind of link in S^3 except alternating links bounds Hopf plumbing?

In a joint work with Goda at Tokyo A & T and Hirasawa at Gakushuin Univ., we proved the following: "Let R be a Seifert surface obtained by applying Seifert's algorithm to an "almost" alternating diagram. Then R is a fiber surface if and only if R is a Hopf plumbing." An almost alternating diagram is a diagram obtained by one crossing change from an alternating one. We moreover confirmed that there is a link with a diagram obtained by two crossing changes from an alternating diagram such that it does not have the same property above.

Harer showed that all fiber surfaces in S^3 can be constructed from the 2-disk by plumbing and deplumbing Hopf bands, and Stallings twists. plumbing is one of Murasugi sum, deplumbing is the reverse operation of plumbing. Stallings twist is defined as Dehn twist along a certain simple closed curve, called a twisting loop, on fiber surface. Then he conjectured that the Stallings twist can be omitted.

I defined a complexity of Stallings twist as the minimal number of connected components of the intersection of a disk bounded by a twisting loop and the fiber surface, and proved that a Stallings twist with complexity 1 is realized by plumbing and deplumbing Hopf bands.

Let M be a closed oriented 3-manifold. One can obtain a new open book decomposition of M by plumbing a positive Hopf band to an open book decomposition of M. This operation is called a positive stabilization of open book decompositions. Giroux showed a one-to-one correspondence between all equivalence classes of open book decompositions of M up to positive stabilization and all isotopy classes of positive contact structure of M. Then, mainly through this one-to-one correspondence, Harer's conjecture mentioned above is solved affirmatively in the homology spheres.

I studied a characterisation of open book decompositions corresponding to overtwisted contact structures based on the one-to-one correspondence, and showed an important role of the twisting loop as the following:

"An open book decomposition corresponds to an overtwisted contact structure if and only if it is equivalent to an open book decomposition with a twisting loop up to positive stabilization."