Research Summary

Sachiko Hamano

The so-called Cousin II problem is finding a holomorphic function with a prescribed zero set. This is not necessarily solvable even in the case of cylindrical domains. We may relax the condition in the Cousin II problem by allowing larger zero, and this relaxed problem is solvable for every Cousin II distribution on a Stein manifold. However, the solution does not tell whether the analytic hypersurfaces defined by the prescribed Cousin II distribution intersect the extra analytic hypersurfaces defined by the solution, and we may investigate the following problem (extra zero problem): given a Cousin II distribution $\mathfrak{D} = \{(U_i, f_i)\}_{i \in I}$ on a complex manifold X, find a holomorphic function f on X such that on each U_i, $h_i := f/f_i$ is holomorphic and $\{x \in U_i \mid f_i(x) = h_i(x) = 0\} = \emptyset$. Such an f is, if exists, called a solution of the extra zero problem for $\mathfrak{D}$. In [2], we studied the solvability of this *extra zero problem*, which is open even in the case of domains in $\mathbb{C}^n$, and affirmatively solved it for every cylindrical domain in $\mathbb{C}^2$. On the other hand, we also solved the extra zero problem in $\mathbb{C}^n$ with $n \geq 3$, negatively, by giving examples of cylindrical domains D in $\mathbb{C}^n$ for every $n \geq 3$ and Cousin II distributions $\mathfrak{D}$ on D for which the extra zero problem is not solvable. As applications of these results, we gave a classification of complex domains, which is a generalization of theorems of Oka and Stein.

In [1], from the viewpoint of convexity in real and complex analysis, we proved a *real analogue* of the Hartogs-Osgood extension theorem in the complex analysis: let D be a domain in $\mathbb{R}^m$ with $m \geq 2$, and K a compact subset of D such that $D - K$ is connected. Then every separately harmonic function on $D - K$ extends to that on D.

In [3], with Professor Hiroshi Yamaguchi, we studied the movement and the variations of Bergman metric and Robin constants on Riemann surfaces as a (plurisubharmonic) functions on Riemann's moduli spaces and Teichmüller spaces: a fibration of pseudoconvex domain by bordered Riemann surfaces is trivial if this movement is pluriharmonic.