Research project

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Project 1 : Classification of knots and links by 5-,7-, and (2,3)-moves

Among those fundamental questions yet to be solved, I study the problem whether or not every link is equivalent to an unlink by n-moves. Montesinous and Nakanishi conjectured that 3- (4- respectively) moves are unknotting operations around 1980, and the conjecture remained unsettled for over 15 years. For these questions, 3-,4-move conjectures are settled for special classes of knots,by skein polynomial invariant, unknotting operations, Fox n-colorings which is deeply related with double branched covering space. In 1984, Nakanishi gave the link L_{2BR} as a candidate of a counterexample to the 3-move conjecture. Later, in 2002, Przytycki and Dabkowski proved that L_{2BR} cannot be reduced to any trivial links by 3-moves by algebraic argument of Burnside group. Askitas gave a candidate of counter example for the 4-move conjecture in 1999. Kawauchi raised another open probrem of 4-move equivalence in 1985, "Is it true that every 2-component link is 4-move equivalent to the trivial link of two components or to the Hopf link?" We find dramatical change for 5-move equivalence. Even the trefoil knot is not 5-move equivalent to trivial links. Since 2004, I have been collaborating with Przytycki and Dabkowski to investigate 5-move equivalence classes of rational links and prime links up to nine crossings (to be completed in [7 (in preparation)]). I will complete the table of 5-move equivalence classes of links. Every 5-move is a combination of two $(2,2)$-moves, however, $(2,2)$-move is known to be an unknotting operations for several classes of links. Harikae-Nakanishi-Uchida conjectured in 1992 that $(2,2)$-move is an unknotting operation. In 2004, Przytycki and Dabkowski proved that the conjecture is not true. As described above, though counterexamples for 3-,$(2,2)$-move conjectures are found, these conjectures hold for several classes of knots and links. Any $(2k+1)$-move is a combination of two $(2,k)$-moves. Przytycki has formulated a problem in 1995, whether any link can be reduced by $(2,3)$-moves to unlink or not. Like 5-move, 7-move which is a combination of two $(2,3)$-moves, is not unknotting operation. I will speculate whether $(2,3)$-moves reduce links to an unlinks, proceed with classification of links by 5-, 7-move equivalence of links, and challenge open problems.

Project 2 : Investigation of Rotation of links

In [5], we showed that the Tristram-Levine signature is invariant under rotations of a link for the case of orientation preserving rotant. I will study the case of orientation reversing rotations and also investigate the behavior of other link invariants under rotaitons. A famous pair of mutant links with 11 crossings, Conway knot and Kinoshita-Terasaka knot, share the same skein polynomial invariants. These knots are distinguished by investigation of their genera, 3 and 2 respectively, by Gabai(1984). It was an interesting observation, because genus is much more rough than skein polynomial invariant as a knot invariant, neverthless it worked for this case. In [5], a key observation was that we can construct 'good' Seifert surfaces which are the Murasugi sum of surfaces of n-rotor part with n-rotational symmetry, and the other part. I will investigate the structures of these Seifert surfaces, from geometrical view points, and examin the genus and how it changes after rotations.