

My results

1. “Seifert complex for links and 2-variable Alexander matrices” :

D. Cooper considered a union of Seifert surfaces of each component of a link. Intersections among Seifert surfaces are allowed, and can be restricted to clasp singularities. He defined it *C-complex*, and investigated especially the case of 2-component C-complexes. We confirmed his method, and gave the characterization of the Alexander matrices for 2-component links. By using the result, we proved the Torres formula, and the Bailey-Nakanishi theorem which is the characterization of the Alexander polynomials for 2-component links with the linking number zero.

2. “Proper link, algebraically split link and Arf invariant” (joint work with A. Yasuhara) :

We defined the new Arf invariant for algebraically split links by using an R-complex. I contributed to show the invariant is well-defined. The ordinary Arf invariant for a proper link is an invariant of the link itself. It satisfies the additivity for boundary links, but does not always satisfy the additivity for algebraically split links. On the other hand, our new Arf invariant for an algebraically split link and an R-complex F is an invariant of the pair of F and a component of F . It satisfies the additivity for algebraically split links with respect to its components.

3. “Component-isotopy of Seifert complexes” :

D. Cooper showed the fundamental moves of 2-component C-complexes. We generalized his result to the case of n -component C-complexes by adding one fundamental move. We also proved this new move cannot be removed by showing an example which are two C-complexes for the Borromean rings. The fundamental moves of singular Seifert surfaces can be obtained by the similar way.

4. “On the additivity of 3-dimensional clasp numbers”:

Let K be a knot in the 3-sphere. The *clasp number* $c(K)$ of K is the minimal number of clasps on clasp disks spanning K . We consider a question “Let K_1 and K_2 be knots. Then $c(K_1 \# K_2) = c(K_1) + c(K_2)$ holds ?” It had been proved that the question is affirmative if $c(K_1 \# K_2) \leq 3$. We proved that the case of $c(K_1 \# K_2) \leq 5$ is affirmative. In the other paper, we showed a table of the clasp number for prime knots with crossing number at most 10.

5. “Detecting non-triviality of virtual links”:

We consider the non-triviality of virtual links by using the *supporting genus* which is the minimal genus of a surface realization of a virtual link diagram. A virtual link has the underlying *flat virtual link* (*projected virtual link*, in the paper) which is obtained by identifying positive crossings and negative crossings. We proved that a virtual link has reduced diagrams by showing an algorithm to obtain a reduced diagram, and two reduced diagrams are related by a finite sequence of Reidemeister III-moves. We can deduce that the supporting genus of a flat virtual link is realized by its reduced diagram, and Kishino’s knot, which had been outstanding whether it is trivial or not, has the supporting genus 2 (*i.e.* non-trivial) by a geometric method. We pointed out that there is a relation between the Virtual Knot Theory and the 2-dimensional hyperbolic geometry.

6. “A classification of closed virtual 2-braids”:

We classified completely closed virtual 2-braids. We obtained conditions by using *surface bracket polynomial* due to Dye and Kauffman, and classified them by using correspondence with a subgroup of 3-string braids.

7. “Connected sum and prime decomposition of virtual links”:

A connected sum of virtual links is not uniquely determined only by virtual link types. We defined *pointed virtual/flat virtual link* which is a pair of a virtual/flat virtual link and finite points on it, and a connected sum of pointed virtual/flat virtual links is determined by the points. By defining equivalence relations appropriately, we defined *prime virtual/flat virtual link*, and proved that prime factors and a connecting tree are uniquely determined.

8. “Reidemeister torsion of homology lens spaces”:

V. Turaev gave a method how to compute the Reidemeister torsion of compact 3-manifolds. In particular, for a homology lens space, it is computed by the Alexander polynomial of a knot in a homology 3-sphere, and a surgery coefficient p/q . By using the result, we consider the case that K has (1) the Alexander polynomial which is the same as a torus knot, (2) the Alexander polynomial of degree 2, and (3) the Alexander polynomial which is the same as a $(-2, m, n)$ -pretzel knot (joint work with Y. Yamada), and conditions that the Reidemeister torsion of a rational surgery along K is the same as that of a lens space. We gave the necessary and sufficient condition for (1), and showed that the Alexander polynomial of K should be $t^2 - t + 1$ for (2). We generalized the results for the case that the Alexander polynomial of K is of degree $2g$. In (3), we confirmed the speciality of the $(-2, 3, 7)$ -pretzel knot. Moreover (4) we characterized the Alexander polynomial of a knot in a homology 3-sphere whose Reidemeister torsion is the same as that of a lens space (joint work with Y. Yamada). (5) We investigate Seifert surgery by using the same technique. We proved that the Alexander polynomial of a knot which has Seifert surgery has informations about multiplicities of singular fibers of the Seifert manifold. (6) We also obtained conditions about lens surgery for 2-component links.

9. “Iwasawa type formula for covers of a link in a rational homology sphere”:

Iwasawa formula by which the order of the ideal class group of a certain branched covering for an algebraic field is computed corresponds to Fox, mayberry-Murasugi, and Porti formula in Knot Theory by which the order of the first homology group of an abelian covering of a homology 3-sphere is computed. We extend slightly the latter formula to the case of a cyclic branched covering of a rational homology 3-sphere. We gave some examples.