

## My plan

Naoko Kamada

Virtual knots are generalization of knots, which correspond to abstract knots. They are useful for studying invariants of links on surfaces systematically. They are also useful for studying quandle homology groups. I would like to do a research on virtual knots and the invariants for classical/virtual knots. After [3] was published, S. Carter, S. Kamada and M. Saito discussed a relationship between virtual knots and the stable equivalence classes of knot diagrams in surfaces in [Scott Carter, Seiichi Kamada and Masahico Saito, *Stable equivalence of knots on surfaces and virtual knot cobordisms*, J. Knot Theory Ramifications vol. 11 (2002), 311–320]. G. Kuperberg proved that a virtual knot is uniquely realized as link diagram on a surface when the genus of the surface is the minimal, [Greg Kuperberg, *What is a virtual link?*, Algebr. Geom. Topol., vol. 3 (2003), 587–591]. By using this result, H. Dye and L. H. Kauffman defined an invariant of virtual knots in [Heather Dye and Louis Kauffman, *Minimal surface representations of virtual knots and links*, Algebr. Geom. Topol., vol. 5 (2005), 509–535]. But in general geometric interpretations of algebraic structures and invariants derived from virtual knot diagrams (the fundamental quandles, the fundamental biquandles, quandle homology invariants, etc), except virtual knot groups, are not well known. I would like to give geometric interpretations of the algebraic structures, and I would like to know the properties of them.

I would like to study the polynomial invariants on virtual knots besides invariants derived from algebraic structures. J. Sawollek generalized the polynomial invariant for knots in a thickened surface defined by F. Jaeger, L. H. Kauffman and H. Saleur in [François Jaeger, Louis Kauffman and Hubert Saleur, *The Conway polynomial in  $R^3$  and in thickened surfaces*, A new determinant formulation, J. Combin. Theory Ser. B, vol. 61 (1994), 237–259] to that for virtual knots in his preprint, [Jörg Sawollek, *On Alexander-Conway polynomials for virtual knots and links*, (1999)]. This is called the JKSS invariant. T. Kishino and T. Satoh [Toshimasa Kishino and Shin Satoh, *A note on non-classical virtual knots*, J. Knot Theory Ramifications, vol. 13 (2004), 845–856], discussed on the JKSS invariant for the virtual knot whose virtual crossing number is one. Such kind of research on invariants for a specific family of virtual knots is quite useful to classify virtual knots. In [Yasuyuki Miyazawa, *Magnetic graphs and an invariant for virtual links*, (2005)], Y. Miyazawa defined a polynomial invariant for a virtual knot. I would like to study his invariant and relationships among the invariants and I would like to construct new invariants.

I am going to construct a table of long virtual knots. Long virtual knot diagrams are immersed line in  $\mathbf{R}^2$  whose end points are fixed and double points are given information on crossings as positive, negative or virtual. In [Vladimir Turaev, *Knots and words*, (2005)] and [Vladimir Turaev *Topology of words*, (2005)] , V. Turaev introduced the notion of “nanoword” which can present long virtual knot diagrams. I would like to construct a table of long virtual knots by using “nanoword”. It may be bigger and more complicated than the table of virtual knot diagrams. In my plan, I am going to make a computer program to list data and calculate invariants.

Some invariants for long virtual knots are defined by generalizing invariants for virtual knots. V. Manturov introduced a quandle for a long virtual knots, in [Vassily Manturov, *Long virtual knots and their invariants*, J. Knot Theory Ramifications, vol. 13 (2004), 1029–1039]. D. S. Silver and S. G. Williams defined the Alexander group for a long virtual knot which is an invariant of algebraic structure [Daniel Silver and Susan Williams, *Alexander groups of long virtual knots*, (2004)]. I would like to study them and construct new invariants for long virtual knots.

I studied a skein module for virtual tangles due to Jones polynomial in the preprint [3]. I would like to study skein modules for virtual knot tangles.

M. Bourgoïn introduced a notion of a twisted link, which is a generalization of a virtual link in 2006. A virtual link is abstracted from a link diagram on a closed oriented surface. On the other hand a twisted link is abstracted a link on a closed surface. I studied a twisted link and gave some presentations. (regional conferences : [16],[17],[18] international : [10]) I would like to study twisted links.

I would like to apply the consequence of these studies for virtual knots to research on classical knots.