Research plan

Yasunari Kurita

Thermodynamics of five-dimensional Kaluza-Klein black holes with squashed horizons

In 2005, Ishihara and Matsuno found that a charged black hole solution in five-dimensional Einstein-Maxwell theory. The horizons of this black hole is topologically S^3 which is an example of Hopf bundle. Therefore, the horizons can be thought of as a twisted S^1 fiber bundle over S^2 base space. Similarly, radial coordinate constant surfaces are topologically S^3, and the aspect ratio of S^1 fiber to S^2 base space is a function of the radial coordinate, that is, it is not unity generally. In this sense, the surfaces can be considered as squashed S^3. The black hole spacetime is not asymptotically flat but asymptotically locally flat. The size of S^1 fiber remains finite even at the spatial infinity and, far from the black hole, that direction is effectively compactified. Due to the twisting of the S^1 fiber, there exists non-trivial gauge field in the Kaluza-Klein reduction. Especially, in no horizon limit, the black hole solution reduces to Kaluza-Klein monopole solution, originally found by Gross and Perry, and Sorkin. It follows that we call the black holes squashed Kaluza-Klein black holes.

It is known that black hole obeys thermodynamic laws. Cai et al. investigated thermodynamic first law of the squashed Kaluza-Klein black holes and indicated that new work term will exist for mass defined by counter-term method. According to their investigation, one of the thermodynamic quantities included in the new work term is a function of a parameter characterizing the squashing of the horizons. However, explicit forms of the new work term and the new thermodynamic quantities were not given by them. In my research, I will study the new work term and thermodynamic quantities and give a thermodynamic formulation for the squashed Kaluza-Klein black holes.

In general relativity, there are many definitions of mass and the black hole takes different values for some different definitions of mass. For example, the Hamiltonian, the Abbott-Deser mass and the Komar mass take different values each other. I will study the thermodynamic first laws for these masses.

After the new thermodynamic quantities are clarified, I will investigate thermodynamic stability of the squashed Kaluza-Klein black hole by obtaining thermodynamic free energy suitable for each thermodynamic environment. Furthermore, the black hole includes charged black string solution in a limit, and thermodynamic stability of the charged black string will also be investigated.

The new thermodynamic quantities are expected to have deep relation with the fact that the horizons are squashed. In my research, the geometrical background of the new quantities will be clarified. Especially, the relation with Misner string will be considered. The situation seems to be similar to the case of rotating black hole having Misner string. I will explain the similarity and the difference from that case.