RESEARCH PROJECTS

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In the future I would like to focus my concentration on the following three researches;

- Around the problem "Is the (quasi-)toric manifold determined by its cohomology ring?".
- Define a class $(M^{4n},\ T^{n+1})$ and study them from the toric topological approach.
- Translate the classical geometric theory into the graph and its application.

All the aboves are included in the toric topology.

The first topic is motivated by Masuda's Theorem "The (quasi-)toric manifold $(M^{2n},\ T^n)$ is completely determined by a $H_T^*(pt)$-algebraic structure of its equivariant cohomology $H_T^*(M)$". From his theorem the answer of our problem might be Yes, but we do not achieve a deep understanding in fact (it is also interesting to find a counter example). As the first test I would like to study manifolds appeared in the paper (5) more. For example the classification of the diffeomorphic types or the relation between the diffeomorphic map of M and the isomorphic map of the cohomology ring $H^*(M)$ will be studied. Since the transitive case is easy, I will consider manifolds in codimension one case. Because diffeomorphic types of their toric manifolds might be solved by improving Choi's study on the classification of complexity one Bott towers, I will consider the relation between the diffeomorphic map of M and the isomorphic map of $H^*(M)$ in codimension one case. Of course in our study it appeared wider class of torus manifolds, so I will study on all of them. I expect to find new side of the toric topology from this research. Final goal is to solve the Masuda's problem. If we could solve this problem, the class (M^{2n}, T^n) (the torus manifold) became to be deeply understood.

The second topic is to find the class which should study as the next of (M^{2n}, T^n) and define it exactly, that is, our goal is to find the class (M^{2m}, T^n) $(m > n)$ such that there exists some interesting combinatorial structure. I think the hypertoric variety in the paper (4) has been studied second to torus manifolds (toric manifolds). This class is denoted by (M^{4n}, T^{n+1}), and its equivariant cohomology ring is described by the combinatorial date of a hyperplane arrangement. I defined hyper torus graph (HTG) by abstracting the graph coming from the hypertoric variety, but HTG contains more interesting classes. For example it contains the graphs defined by $S^4 \times \mathbb{C}^2$, $\mathbb{H}P(n)$ or Q_{2n}. In some sence their manifolds are connected by HTG. So the first aim of this research is to find their geometric meaning and I will study a natural combinatorial structure which corresponds with them (not HTG). An equivariant cohomology of some torus manifold can be calculated by a combinatorial structure of a manifold with corner. On the other hand the hyperplene arrangement is so for the hypertoric variety. Hence the class defined from this research might correspond with combinatorial object which generalizes the hyperplane arrengement. Then I also study what the quaternionic torus manifold is.

I do not know what lies in the third topic (there might be an interesting things or not so). The motivation of this study is to translate the symplectic geometry into graphs by Guillemin-Zara and to be able to consider HTG as the tangent bundle of GKM graphs. Because we can consider HTG as the tangent bundle of GKM graphs, I think there are many geometric theories translated into graphs. If not so there also many such theories as Guillemin-Zara. We might be able to evolve the cobordism theory on graphs. In the first step of this research it might be only the translation work and there might not be interesting mathematics, but Guillemin-Zara found the application to the combinatorial theory finally. I expect there might be such surprising things in this research. And interesting things might be found from the progress of this translation.

The above three things, Masuda's problem, quaternionic torus manifold, translation into graphs are central problems in the toric topology, so I will solve these problems.