SUMMARY OF MY RESEARCH

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I have researched transformation groups on manifolds from topological point of view. My research can be divided into the following three researches. The numbers which appear in the followings correspond with the numbers of "1.Papers" in the "List of my paper".

0.1. **Classification of the compact Lie group action.** In the paper (2), I completely classified compact Lie groups, which act on a rational cohomorogy complex quadric M (a manifold whose rational cohomology is isomorphic to Q_{2n}) with codimension one principal orbits, and I also classified topological types of M. To classify it, I used the Uchida's method introduced in 1978. I proved there is exactly one manifold whose topological type is not the complex quadric, and one group action on the complex quadric which did not get from the research of A. Kollross in 2002 (he classified K, which is a subgroup of G and acts on G/H with codimension one orbits). Moreover as one of applications of the Uchida's method, it is easy to see that the relation $\mathbb{C}P(n) \cong Q_n/\mathbb{Z}_2$ holds between the general complex quadric Q_n and the complex projective space $\mathbb{C}P(n)$, to compare each orbits.

0.2. **Smooth non-compact Lie group action.** In the paper (1), I constructed infinitely many smooth $SL(m, \mathbb{H}) \times SL(n, \mathbb{H})$-actions on $S^{4(m+n)-1}$ to construct smooth $\mathbb{R}^2$-actions on S^7.

In the paper (3), I used the fact that the quotient space of $\mathbb{C}P(2)$ by complex conjugation is 4-sphere S^4 and constructed a continuous $SL(3, \mathbb{R})$-action on S^4 whose restricted $SO(3)$-action is the conjugated action. Moreover I proved $\mathbb{C}P(2)/\mathrm{conj} \cong S^4$ directly by using the application of the above Uchida's method

0.3. **Toric topology.** Toric topology is the study of algebraic, combinatorial, differential, geometric, and homotopy theoretic aspects of a particular class of torus actions, whose quotients are highly structured.

The paper (4) corresponds with the study of combinatorial aspect. I defined a hyper torus graph (HTG) as a new class of graph which is similar to the GKM graph and studied their equivariant cohomology ring structure. I abstract graphs defined by a hypertoric variety or a cotangent bundle of a toric manifold, and defined HTG. As the character of this graph, there are not only edges but also legs (an out going half line from one vertex). Because of this character we can use geometric ideas for graphs. For example we can define a vector bundle of GKM graph to attach legs on each vertex, and we can also define a neighborhood of subgraphs. In this paper I defined the Mayer-Vietoris exact sequence of graphs and applied it to show the equivariant cohomology of some HTG is isomorphic to a ring defined by the combinatorial date on HTG.

The paper (5) corresponds with the study of geometric (transformation theoretic) aspect. Moreover this study corresponds with the research 0.1. I classified the pair a rank n compact Lie group G and a torus manifold (M, G) which is transitive or has codimension one orbits and whose restricted action (M^{2n}, T^n) is a torus maniofld. The torus manifold was defined by Hattori-Masuda in 2003 and is a minifold which has an effective half dimensional torus T^n-action. In the toric topology, this class is the most studied class. I proved the transitive case is just the product of complex projective spaces $\mathbb{C}P(n)$ and even dimensional spheres S^{2m} from the classification result of compact Lie groups, the codimension one case is just the $\mathbb{C}P(k)$ (or S^{2l}) bundle over the product of $\mathbb{C}P(n)$ and S^{2m} from the Uchida's method. The difference form the paper (2) is to consider the family of manifolds (torus manifolds). In the paper (2) we fixed the cohomology ring of manifold. So we see the Uchida's method is useful not only to classify the case fixed a cohomology ring but also to classify some family of manifolds.