Results of my research

Hiromasa Moriuchi

A link is n circles $S^1 \cup \cdots \cup S^1$ embedded in a 3-sphere S^3. As a generalization of classical knot theory, we can consider other objects embed in S^3. A spatial graph is a graph embedded in S^3. If the graph consists of two vertices and three edges such that each edge joins the vertices, then it is called a θ-curve. Moreover, if the graph consists of two loops and an edge jointing the vertices of each loop, then it is called a handcuff graph.

In knot theory, there exists the study of creating a prime knot table. Similarly, there exists the study of making a table of spatial graphs. In particular, some researchers worked on making a θ-curve table. J. Simon made a table of θ-curves with up to five crossings in 1987, R. A. Litherland announced a table of prime θ-curves with up to seven crossings in his letter of 1989, T. Harikae enumerated special θ-curves with up to nine crossings in 1987. However, we can suppose that their θ-curves obtained by adding an edge to some knot. Moreover, the completeness of Litherland's table had not been proved. Then I would like to complete the table of prime θ-curves with up to seven crossings.

I applied Conway's method to enumerate θ-curves. In 1969, J. H. Conway made an enumeration of prime knots with up to eleven crossings by introducing the concept of a tangle and a basic polyhedron. Here, a tangle is a pair embedded in a 2-sphere S^2 consisting of a 3-ball B^3 and a (possibly disconnected) proper 1-submanifold t with $\partial t \neq \emptyset$, and a basic polyhedron is the 4-regular planar graph which has no bigon. We can obtain knots from basic polyhedra by substituting tangles for their vertices.

In order to apply Conway's method, we need the following works. First, I enumerated algebraic tangles with up to seven crossings (see [3]). Second, I constructed a prime basic θ-polyhedron to enumerate prime θ-curves. Here, a θ-polyhedron is a connected planar graph embedded in 2-sphere, whose two vertices are 3-valent, and the others are 4-valent. Then our θ-polyhedron is different from Conway's polyhedron. Since I would like to make a prime θ-curve table, I omitted a non-prime θ-polyhedron. Then there exist twenty-four prime basic θ-polyhedra with up to seven 4-valent vertices. We can obtain θ-curves from prime basic θ-polyhedra by substituting tangles for their 4-valent vertices.

In master's thesis [1], I obtained all the prime θ-curves with up to seven crossings, which are the same table as Litherland's. Litherland classified these θ-curves by constituent knots and the Alexander polynomial. I classified them by the Yamada polynomial (see [6]).

Moreover, I first enumerated all the prime handcuff graphs with up to seven crossings in similar way. I also classified these handcuff graphs by using the Yamada polynomial (see a list of my papers [4]).