Plan of my research

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To find invariants of knots is one of the goals of knot theory. An invariant takes the same value on equivalent knots. In [1][2], to classify spatial graphs by some local moves, I used invariants which do not change by the local moves. However invariants can be used to show that two objects are different. I thoughtthat other information can be obtained by examining the degree of the differences. Paying attention to the situation where invariants change, I investigate what information the differences have. From this view point, I try to classify braids by exchange moves. I plan the following researches:

(1) On an infinite sequence of mutually non-conjugate braids which close to the same knot

By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed n-braid ($n = 1, 2$ or 3) has only a finite number of conjugacy classes of n-braid representatives in the n-braid group B_n. Moreover, Birman-Menasco proved that if an oriented link has infinitely many conjugacy classes of n-braid representatives in B_n, the infinitely many conjugacy classes divide into finitely many equivalence classes by exchange moves. For the unknot (or $(2, p)$-torus link) K, an infinite sequence of 4-braids which closed to K in which all braids fall into a single equivalence class under the equivalence relation generated by exchange moves have already been given. If other knots have such infinite sequences, I thought that it was effective to examine, for instance, how the exchange moves change the conjugacy invariants of braids and the knot invariants of the axis-addition links and how many classes include infinite sequences in order to give the classification by exchange moves. Hence I tried to construct such infinite sequences. I started from examining the Conway polynomial of the axis-addition links of the braids in the existing infinite sequences experimentally by using Kodama Knot Program, and I found the rule in the coefficients of the Conway polynomial of these axis-addition links. Therefore, to show that braids are non-conjugate, I evaluated the difference of the coefficients of the Conway polynomials of the axis-addition links of braids.If some other local moves and invariants also can be used for this method, it seems that constructing infinite sequences of other affiliates become possible. By examining the braids in existing sequences, I would like to find other invariants which I can use for this method. Also by examining the influence of exchange moves on conjugacy invariants of braids and the influence on knot invariants of the axis-addition links, I would like to find the clue of the classification by exchange moves.

(2) On irreducibility of braids

H. R. Morton gave the first example of irreducible 4-braid which closes to the unknot. Later, from Morton's example, T. Fiedler constructed an infinite sequence of mutually non-conjugate irreducible 4-braids which close to the unknot. However, in general, it is difficult to show the irreducibility of braids. Hence Morton's and Fiedler's arguments in the proof of irrecucibillity of braids remains restricted to 4-braids, and the generalizations to higher braid groups has not been known yet. In [4], for a knot of even braid index $n > 3$ satisfying a certain conditions, I gave an infinite sequence of mutually non-conjugate n-braids which close to the knot. Since all braids in my sequence realize the braid index of the knot, they are irreducible. In the point that they are irreducible, it can be said that the sequence is a kind of extension of Fiedler's sequence. However Morton's and Fiedler's proofs of the irreducibility are greatly different from mine. To understand the influence of the stabilizations and destabilizations on the conjugacy classes in the braid group, it seems that to examine irreducible braids is effective. Since an effective method to detect irreducibility has not been found yet, I would like to find it from the viewpoint of knot theory.