# Results of my research

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In [1], we gave a homology classification of spatial graphs. K. Taniyama showed that two spatial embeddings are spatial-graph-homologous if and only if they have the same Wu invariant. We showed that two spatial embeddings of a graph are spatial-graph-homologous if and only if all of their linking numbers and Simon invariants coincide. Therefore Homology classification came to be given by the simple calculation, since both linking number and Simon invariant are integral invariants that are easily calculated from a regular diagram of a spatial graph. T. Motohashi-Taniyama showed that two spatial embeddings are spatial-graph-homologous if and only if they are delta-equivalence and Taniyama-A. Yasuhara showed that a delta-move does not change any order 1 finite type invariant of spatial graphs. Therefore it follows that linking number and Simon invariant determine all of order 1 finite type invariants of spatial graphs.

A spatial embedding of graph $G$ is said to be locally unknotted, if there is a set of cycles $\Gamma$ of $G$ which forms a basis for $H_1(G;\mathbb{Z})$ and the set of knots in the spatial embedding corresponding to $\Gamma$ bounds a set of interior-disjoing disks. A locally unknotted spatial embedding of a graph was introduced by K. Kobayashi. In [3], I got an idea from his concept and defined a collection of spanning surfaces of a spatial graph. We considered a set of connected, compact and orientable interior-disjoint surfaces bounded by knots in a spatial graph. If each surface has a distinct boundary, we say that the set is a collection of spanning surfaces. Especially if each surface is homeomorphic to a disk, the set is called a collection of spanning disks. T. Endo-T. Otsuki proved that any graph has a locally unknotted spatial embedding. However, in general, the rank of $H_1(G;\mathbb{Z})$ is not an upper bound of the number of spanning surfaces. Hence I gave the upper bound of the number of spanning surfaces and showed that this upper bound is the least upper bound by constructing a spatial embedding which realizes the upper bound with disks.

In [2], we tried to extend the concept of boundary links to spatial graphs. We defined a boundary spatial embedding of a graph $G$ as a spatial embedding of $G$ which has a collection of spanning surfaces with respect to the set of all knots in the embedding. From the result in [3], we see that not every graph has a boundary spatial embedding. Hence we gave a characterization of graphs which have boundary spatial embeddings. Then we classified boundary spatial embeddings of a graph completely up to self pass-equivalence and showed that any two boundary spatial embeddings of a graph are self sharp-equivalent. These are natural extensions of the results concerning boundary links given by L. Cervantes-R. A. Fenn and T. Shibuya.

In [4][6][7], using some knot invariants, I tried to extend existing results concerning conjugacy classes of braids. By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed $n$-braid ($n = 1, 2$ or $3$) has at most three conjugacy classes of $n$-braid representatives in the $n$-braid group. However $n \geq 4$, the situation changes. An infinite sequence of mutually non-conjugate 4-braids representing the unknot (resp. $(2,p)$-torus link) has already been discovered. In [6][7], generalizing their sequence, for any knot $K$ (or a link satisfying certain conditions) represented as a closed $n$-braid ($n \geq 3$) I gave an infinite sequence of mutually non-conjugate $(n+1)$-braids representing $K$. In [4], I generalized the infinite sequence of mutually non-conjugate irreducible 4-braids representing the unknot constructed by Fiedler and gave such an infinite sequence of irreducible braids for some knots. However, it is difficult to extend Fiedler's argument to higher braid group. Therefore, we used the braid index of knot to show the irreducibility of braids.In existing results, to prove that braids in the sequence are mutually non-conjugate, the homomorphism from the 4-braid group to the 3-braid group and a certain coujugacy invariant were used. On the other hand, I used Conway polynomial which is a knot invariant. I evaluated third coefficient of the Conway polynomial of the axis-addition links of the braids in my sequence by using the linking numbers. Hence the proof can be given by simple calculations.