The plan of our study

Every link can be realized as a closed braid. A. Kawauchi suggested the following project: Enumerate the prime links by using the braid expressions and enumerate the 3-manifolds by using the link table. Our main purpose is to complete the table of 3-manifolds according to this project.

We describe the outline of the project. A well-order (called a *canonical order*) was introduced in the set of links by A. Kawauchi [K] (see also A. Kawauchi and I. Tayama [KT]).

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link and we call the number the *length* of the link. We note that a link L is smaller than a link L' in the canonical order if the length of L is smaller than that of L', and for any natural number n there are only finitely many links with lengths up to n. Let $\mathbf{L}^p$ be the set of prime links and $\mathbf{M}$ the set of closed connected orientable 3-manifolds. Let $\chi : \mathbf{L}^p \to \mathbf{M}$ be a map defined by $\chi(L) = \chi(L, 0)$ (this means the result of the 0-surgery on S^3 along L). Then it is known that χ is surjective and A. Kawauchi defined a map $\alpha : \mathbf{M} \to \mathbf{L}^p$ by $\alpha(M) = \min\{L \in \chi^{-1}(M) :\ L' \in \chi^{-1}(M), \pi_1(E(L)) = \pi_1(E(L')) \Rightarrow L < L'\}$ for $M \in \mathbf{M}$, where $E(L)$ is the exterior of L. By using α, we consider $\mathbf{M}$ as a subset of $\mathbf{L}^p$ and introduce the well-order into $\mathbf{M}$.

We enumerated the links with lengths up to 10. The next step is to enumerate the link exteriors and we completed the table of link exteriors with lengths up to 9. We are now making the manifold table with length up to 9. After that we will extend our exterior table and manifold table with up to length 10.

References

[K] A. Kawauchi, A tabulation of 3-manifolds via Dehn surgery, Boletin de la Sociedad Matematica Mexicana (3) 10 (2004), 279–304.

[KT] A. Kawauchi and I. Tayama, Enumerating prime links by a canonical order, Journal of Knot Theory and Its Ramifications Vol. 15, No. 2 (2006) 217–237