Summary of previous study

I.Hoop conjecture in five-dimensions

In the context of the brane world scenario, which is a higher-dimensional universe model inspired by the string theory, matter fields and gauge fields are confined on the brane and only the gravitational field can propagate to the extra-dimensions. In this model, it is suggested that the mini black holes would be produced in a future linear collider. In higher-dimensional spacetime, "hyper-hoop conjecture" has been proposed as the criterion for black hole production. According to this conjecture, it is necessary and sufficient for black hole production that a mass M concentrates on sufficiently small region. This criterion is expressed by using the $(D-3)$-dimensional volume of a $(D-3)$-dimensional submanifold which encircle the mass M. We analyze initial data in five-dimensional spacetimes in which a four-dimensional spheroid exists as source of the gravitational field and obtain consistent results with the hyper-hoop conjecture. In addition, our results suggest a possibility of naked singularity formation by the spindle gravitational collapse in five-dimensional space-time.

II. Gravitational lensing of gravitational waves in a clumpy universe

1. Inhomogeneity in our universe

Many cosmological observations suggest that our universe is homogeneous and isotropic in the scale of ten million light-year. In contrast, there is a lot of inhomogeneities such as stars, galaxies and so on in smaller scale than ten million light-year. One of the most important subject in the present observational cosmology is quantitative evaluation of the inhomogeneity. Then, we study a way of investigation into the inhomogeneity using gravitational lensing of gravitational waves.

2. Gravitational waves from a binary system

The size of a compact binary system is much smaller than a galaxy or cluster of stars. The light ray or gravitational waves from such a small source are easy to be affected by the small scale inhomogeneity, and we can extract the information about the inhomogeneity from this effect. In addition, it is the other feature of binary systems that the distance to the system can be estimated by the observed amplitude in high accuracy. Therefore, combining that with the redshift given by the observation of the parent galaxy, we can obtain the distance-redshift relation.

3. Effects of gravitational lensing on the distance-redshift relation

A beam of gravitational waves is converge and amplified by the effects of gravitational fields of astronomical objects which are located near the ray. Scattered waves are superposed and interfere with each other. These effects depend on the magnitude relation of the wavelength λ and gravitational radius l. If the amplitude varies due to the lensing effects, the observed distance varies. Then, we numerically simulated the gravitational lensing effects on the distance-redshift relation. Here, we assume that all of the matter in the universe takes the form of uniformly distributed point masses with the mass M. We have numerically calculated the observed distances of a number of sources in the case of $\lambda \ll l$ and $\lambda \gg l$.

In the short-wavelength case, $\lambda \ll l$, the dispersion of the distance becomes large. The geometrical optics approximation is valid in this case, and the direction of the rays are bent by lensing effects. Then, there are several paths between the source and the observer. Since rays which come from each path interfere with each other at the observer, the amplitude depends on where the observer is placed on the interference pattern. This is the reason for the large dispersion.

In the long-wavelength case, $\lambda \gg l$, the distances-redshift relation for the fully homogeneous and isotropic universe is reproduced with small distance dispersion. Because, the waves are hardly influenced by lenses.

These results suggest that we might use it to gain information about the typical masses of lens objects.