Research Plan

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Quantum topology for covering spaces. Building on our surgery presentations for (irregular) dihedral covering spaces and for dihedral covering links, Andrew Kricker and I would like to improve the construction of a dihedral analogue of the rational Kontsevich invariant from my thesis. In particular we would like to prove a loop expansion property for this invariant which allows it to be viewed as a D_{2n}-equivariant LMO invariant.

Specializing to integral homology 3–spheres, we would then like to lift Cappell and Shaneson's formula for the Rohlin invariant of a branched dihedral covering space to a formula for the Casson invariant. We would like to do this as Garoufalidis and Kricker did in the abelian case, calculating 'residues' of the 2–loop part of the invariant, and 'normalizing' by values at unknots (in the abelian case this corresponds to correcting by the signature function of the knot).

Pushing further, I would like to understand the dihedral analogues of the Alexander polynomial and the 2–loop polynomial more deeply. In particular, I would like to calculate the new invariants for knots up to some number of crossings, and investigate whether they assume a special form if the knot is slice or if the covering space is a Seifert fibred space. Based on arguments of Garoufalidis and Levine, I expect that the dihedral rational Kontsevich invariant, and hopefully also its 1–loop part, should relate to the Cochran–Orr–Teichner filtration of knot concordance group.

Understanding the surgery picture for A_4. Now that I can translate from A_4 covering presentations to surgery presentations, I would like to know more about A_4 coverings and covering links. So far nobody has worked on this as far as I know. Initial questions include classifying 3–manifolds which arise as A_4 covering spaces, and finding and understanding simple numerical properties of the linking matrix of the covering link.

To prove an analogue of the Przytycki–Sokolov–Sakuma result about descent of symmetries of a covering space to symmetries of some surgery presentation, we should consider A_4 covering spaces branched over (boundary? algebraically split?) links. I do not yet know how to answer the most basic question— whether there exist links with A_4 symmetries, and how to classify them.

A useful proof for Kirby-like theorems for covering spaces. Our surgery presentations for branched dihedral covering spaces are unique up to Kirby moves, which descend to an extended set of 'Kirby moves' on surgery presentations for the coloured knot (K, ρ) 'downstairs'. I originally proved this using Fenn and Rourke's 1979 version of the proof of Kirby's theorem, as Garoufalidis and Kricker proved the analogous statement for knots. I'm working with Kricker to reprove our respective 'Kirby–like theorems' by generalizing Matveev and Polyak's more direct and more recent proof of the Kirby theorem. This may give a 'machine' to prove a more general class of theorems of this type, including for A_4 coverings and for slice knots.

Yoshida's abelianization of the $SU(2)$ WZW model. Now that I have understood Yoshida's parametrization (with Hansen), the next step is to understand how he pulls back holomorphic sections from the Prym variety. The map from the Hitchin moduli space to the Prym variety is a simple branched cover, and the question is how sections behave in the vicinity of the branch locus. Hansen and I have understood the branching locus as a theta divisor (based on Yoshida's argument), and based on this I am working to reconstruct the point–inverse vector field from a space of Prym varieties to a universal cover of the space of double-covers of the Riemann surface, filling in a gap in Yoshida's proof pointed out by Teleman. I would then like to extend Yoshida's invariant to a full TQFT, for which Φ may have poles.

Once the TQFT is constructed, I would like to prove that it is equal to the Reshetikhin–Turaev TQFT. To prove this is to make the correct choice of gluing axiom for the TQFT (so as to agree with Grove's version of the Reshetikhin–Turaev TQFT axioms), and to prove that the invariant satisfies this axiom. Next, using this invariant I would like to recover calculations of Rozansky, Hansen, and Takata for quantum invariants for Seifert manifolds, and of Andersen and Hansen for surgeries on the complement of the figure 8 knot. This work shows how technical such calculations can be when the RT–approach is used, and recovering these results from Yoshida's approach (and maybe getting more stronger results) will give some measure of just how much of the technical calculation this more direct and geometric approach cuts away.