a Braided homology

In the PhD thesis and satellite papers, I constructed general *homology and representation theories* for vector spaces endowed with a *braiding* (= a Yang-Baxter operator). It generalizes the work of Carter-Elhamdadi-Saito and Eisermann. Further, I associated *"structural" braidings* to various algebraic structures (associative and Lie/Leibniz algebras, quandles/racks, etc.) in such a way that the YBE (= Yang-Baxter equation) for these braidings is equivalent to the defining relation for the original structure. E.g., the associativity can be seen as the YBE for $\sigma(v \otimes w) = 1 \otimes v \cdot w$. My homology and representation theories for the "structural" braidings above turn out to include the usual theories for the corresponding structures (e.g., Hochschild or quandle/rack homologies). This flow of ideas can be presented as follows:

algebraic structure $\overset{\text{case by case}}{\rightsquigarrow}$ **braiding** $\overset{\text{a general theory}}{\longmapsto}$ braided homology

I thus include a new step to the usual "structure $\mapsto$ homology" scheme. This creates a *unifying framework* for a study of different algebraic structures and their homologies, explaining their similarities. Such similarities were investigated for the pair associativity/self-distributivity by J. Przytycki.

Two generalizations of the above constructions were effectuated:

- ✓ Everything still work in a *preadditive monoidal category* $\mathcal{C}$, giving for free dual and super versions of my results. Moreover, one gets an inclusion of the categories of certain structures (e.g., associative) into the category of braided objects in $\mathcal{C}$.
- ✓ Considering multi-term *braided systems* and their braided homologies, one recovers homology theories for bialgebras, Hopf (bi)modules, Yetter-Drinfel′d modules, etc.

b Generalized self-distributivity

SDS (= self-distributive structures) are sets endowed with a binary operation which is distributive with respect to itself and satisfies some additional conditions (leading to the notions of quandle, rack, shelf, etc.). They are enjoying a rising popularity due to recent applications to knot theory and Hopf algebra classification. I develop a new method of defining *GSDS (= generalized self-distributive structures) in a symmetric category*. My approach generalizes that of Carter-Crans-Elhamdadi-Saito. Surprisingly, it allows to interpret *associative, Lie and Hopf algebras as GSDS*. Since the usual braiding for SDS lifts to the categorical level, my braided homology theory is applicable to GSDS.

c Virtual braid groups as Hom-sets

The topological part of my work revolves around *virtual knot theory*, initiated by L. Kauffman. In particular, I proposed a categorical interpretation of *virtual braid groups* (defined by V. Vershinin and related to virtual knots by S. Kamada) as Hom-sets of a certain *free doubly-braided category*. Applications include a new interpretation of V.O. Manturov's *virtual racks* and of the *twisted Burau representation* of Silver-Williams. This categorification is different to that of Kauffman-Lambropoulou.