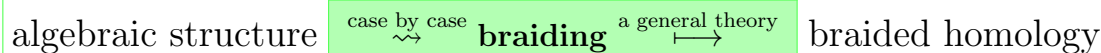


Braided homology

In the PhD thesis and satellite papers, I constructed general *homology and representation theories* for vector spaces endowed with a *braiding* (= a Yang-Baxter operator). It generalizes the work of Carter-Elhamdadi-Saito and Eisermann. Further, I associated “*structural*” *braidings* to various algebraic structures (associative and Lie/Leibniz algebras, quandles/racks, etc.) in such a way that the YBE (= Yang-Baxter equation) for these braidings is equivalent to the defining relation for the original structure. E.g., the associativity can be seen as the YBE for $\sigma(v \otimes w) = 1 \otimes v \cdot w$. My homology and representation theories for the “structural” braidings above turn out to include the usual theories for the corresponding structures (e.g., Hochschild or quandle/rack homologies). This flow of ideas can be presented as follows:



I thus include a new step to the usual “structure \mapsto homology” scheme. This creates a *unifying framework* for a study of different algebraic structures and their homologies, explaining their similarities. Such similarities were investigated for the pair associativity/self-distributivity by J. Przytycki.

Two generalizations of the above constructions were effectuated:

- ✓ Everything still work in a *preadditive monoidal category* \mathcal{C} , giving for free dual and super versions of my results. Moreover, one gets an inclusion of the categories of certain structures (e.g., associative) into the category of braided objects in \mathcal{C} .
- ✓ Considering multi-term *braided systems* and their braided homologies, one recovers homology theories for bialgebras, Hopf (bi)modules, Yetter-Drinfel’d modules, etc.



Generalized self-distributivity

SDS (= *self-distributive structures*) are sets endowed with a binary operation which is distributive with respect to itself and satisfies some additional conditions (leading to the notions of quandle, rack, shelf, etc.). They are enjoying a rising popularity due to recent applications to knot theory and Hopf algebra classification. I develop a new method of defining *GSDS* (= *generalized self-distributive structures*) in a *symmetric category*. My approach generalizes that of Carter-Crans-Elhamdadi-Saito. Surprisingly, it allows to interpret *associative, Lie and Hopf algebras as GSDS*. Since the usual braiding for SDS lifts to the categorical level, my braided homology theory is applicable to GSDS.



Virtual braid groups as Hom-sets

The topological part of my work revolves around *virtual knot theory*, initiated by L. Kauffman. In particular, I proposed a categorical interpretation of *virtual braid groups* (defined by V. Vershinin and related to virtual knots by S. Kamada) as Hom-sets of a certain *free doubly-braided category*. Applications include a new interpretation of V.O. Manturov’s *virtual racks* and of the *twisted Burau representation* of Silver-Williams. This categorification is different to that of Kauffman-Lambropoulou.